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(6) QUALITY CONTROL CONSIDERATIONS FOR FABRICATION OF MODEL PROPELLERS INTENDED FOR CAVITATION TESTING.

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Subject: Quality Control Considerations for Fabrication of Model Propellers Intended for Cavitation Testing

References: See page 23

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NOMENCLATURE

c	profile chord length
C_p	pressure coefficient, $(p - p_o)/ \frac{1}{2} \rho V_o^2$
$C_p\|_{min}$	minimum pressure coefficient
C_{p_s}	pressure coefficient on blade surface
C_ℓ	section lift coefficient
C_{ℓ_i}	section lift coefficient at ideal attack angle (design lift coefficient)
E	relative error in cavitation number caused by blade surface asperity, $(\sigma_{cR} - \sigma_{cs})/\sigma_{cs}$
f_M	maximum camber of blade profile
h	maximum camber of profile or maximum height of surface asperity
k_c	camber correction factor
Re	Reynolds number based c blade section chord
V_o	free stream velocity
v_*	skin friction velocity, $\sqrt{\tau_o/\rho}$
x	distance along chord line
y	distance normal to chord line
α	angle of attack measured with respect to the chord line
α_i	ideal or "design" angle of attack
$\overline{\beta}$	mean relative inflow angle measured with respect to propeller plane
$\pm\Delta\beta$	variations in propeller inflow angle caused by hull-induced inflow distortions
δ	boundary layer thickness
δ_1	displacement thickness

δ_2	momentum thickness
δ_ℓ	viscous sublayer thickness
ε	dimensionless bump height h/ℓ
$\eta(x)$	ordinate of bump
ν	kinematic viscosity
ρ	fluid density
ℓ	length of limb
σ	cavitation number, $(p_o - p_v)/ \frac{1}{2} \rho V_c^2$
σ_{cs}	inception cavitation number on surface of a smooth blade
σ_{cR}	incipient cavitation number on roughened body or on body with asperity
σ_{fp}	cavitation number of an asperity on a flat plate

INTRODUCTION

Experience with model propeller tests in cavitation tunnels and towing tanks seems to suggest that quality control requirements with respect to blade-surface contours and roughness may be more severe for cavitation onset measurements than is the case for overall propulsive performance evaluation. Therefore, the 16th ITTC Cavitation Committee has been assigned the task of determining whether or not sufficient basic data are now available to enable the formulation of quality control recommendations which member organizations can use when fabricating propeller models for cavitation testing. This report documents one aspect of a more comprehensive review of the subject carried out by the committee upon the effects of blade surface roughness and contour irregularities on cavitation inception.

The committee had agreed that the present study should be carried out for a propeller such as that of the "Sydney Express" for which observations of cavitation have been made on the full scale propeller at sea and at model scale in various laboratories. However it was found that the propeller fitted to the "Sydney Express" has blades with sharp leading edges. This geometry can produce laminar separation or laminar bubbles on the blade leading edges. Since analysis methods are not generally available for such noncavitating flows it was decided to use a different example for the present calculations. Accordingly, the skewed propeller fitted to one of the San Clemente OBO class merchant ships was selected.

The design parameters for this propeller are documented in DTNSRDC report P-500-H-04 [1, 2]*. Only a few of the characteristics given in this

*Numbers in brackets indicate references cited below.

source are required for the present calculations. Moreover, Reference [1] summarizes towing tank results on the inflow velocities in the plane of the propeller at the stern of the hull. Therefore estimates for the relative inflow velocity and blade element attack angles are also available. These data provide the basic ingredients needed for the present work in which a blade element at the .7 radius point is taken to be representative of the hydrodynamics of the propeller. Only blade surface cavitation is considered.

PRELIMINARY CALCULATIONS

The full scale OBO propeller has a radius of 13 ft. The blade chord at .7 radius is 6.557 ft. The towing tank model has a radius of 4.333 in. and at .7 radius the blade chord is 2.186 in. The factor between full scale and model is 36.

Reference [1] gives the average relative inflow angle $\overline{\beta}_i$ as 11.72° at the .7 radius station. If we assume a ship speed of 17 knots (28.39 fps) full scale and 3 knots (≈ 5 fps) for towing tank tests we can use this information to estimate the Reynolds number range for the present calculations. If we base the Reynolds number on blade element chord length and relative inflow speed we find for the model that $Re \doteq 3 \times 10^5$. For the full scale propeller we find $Re \doteq 9 \times 10^7$. For the full scale ship the speed of 17 knots is close to its maximum speed. Therefore we should consider a range of Reynolds numbers in order to allow for lesser speeds. In the case of the model we need to consider a range of Reynolds numbers which will permit one to consider towing tank tests and at higher speeds cavitation tunnel tests. As a result of these considerations we have taken the following range of Reynolds numbers for the present work.

Re for model tests	Re for full scale
3×10^5	10^7
10^6	3×10^7
3×10^6	10^8

Turning now to blade element geometry at the .7 radius point, we note that a NACA series 6 camber line with a = 0.8 was combined with a modified thickness distribution [3] in order to obtain a typical blade section. The modified profile thickness distribution is given in terms of percent chord as follows.

x/c	y/c	x/c	y/c	x/c	y/c
0	0	20	3.9957	65	4.3992
.5	.6648	25	4.3629	70	4.0545
.75	.8120	30	4.6382	75	3.6303
1.25	1.0436	35	4.8325	80	3.1265
2.5	1.4663	40	4.9527	85	2.5437
5.	2.0655	45	5.000	90	1.8828
7.5	2.5253	50	4.9635	95	1.1453
10.	2.9073	55	4.8525	97.5	0.7485
15.	3.5213	60	4.6649	100.	0.3333

According to Abbott and von Doenhoff [4], the equation of the mean-line for series 6 profiles is

$$\frac{y}{c} = \frac{C_{\ell_i}}{2\pi(1+a)} \left\{ \frac{1}{1-a} \left[\frac{1}{2}\left(a - \frac{x}{c}\right)^2 \ln\left|a - \frac{x}{c}\right| - \frac{1}{2}\left(1 - \frac{x}{c}\right)^2 \ln\left(1 - \frac{x}{c}\right) + \frac{1}{4}\left(1 - \frac{x}{c}\right)^2 - \frac{1}{4}\left(a - \frac{x}{c}\right)^2 \right] - \frac{x}{c} \ln\frac{x}{c} + g - h\frac{x}{c} \right\} ,$$

where

$$g = -\frac{1}{1-a}\left[a^2\left(\frac{1}{2}\ln a - \frac{1}{4}\right) + \frac{1}{4}\right] \quad ,$$

$$h = \frac{1}{1-a}\left[\frac{1}{2}(1-a)^2 \ln (1-a) - \frac{1}{4}(1-a)^2\right] + g \quad .$$

The ideal attack angle is

$$\alpha_i = \frac{C_{\ell_i} h}{2\pi(a+1)} \quad ,$$

where h is the maximum camber of the blade section as a fraction of its chord. In the present calculations we put a = .8 and eventually combined the mean line with the above thickness distribution in order to obtain the profile ordinates. However before this could be done it was necessary to convert the .7 radius profile for the OBO propeller into an equivalent two-dimensional section by removing the induction effects which were necessary because of lifting surface considerations in the propeller design.

On page 402, Abbott and von Doenhoff, the data listed for the mean line having a = 0.8 show that if $C_{\ell_i} = 1.0$, $\alpha_i = 1.54°$ and the camber is

$$\left.\frac{f_M}{c}\right|_{2-D} = .0679 \quad .$$

From Table 10 on page 38 of Reference [1] we find at .7 radius that

$$\frac{f_M|\text{due to loading}}{f_M|_{2-D}} = k_c = 2.025 \quad .$$

at Table 7 of the same report shows that $\left.\frac{f_M}{c}\right|$ due to loading is .0323 at .7 radius. Therefore,

$$\left.\frac{f_M}{c}\right|_{2D} = \frac{.0323}{2.025} = .01595 \quad .$$

moreover, $\left.\frac{f_M}{c}\right|_{2\text{-}D} = .0679\ C_{\ell_i}$ from which it follows that

$$C_{\ell_i} = \frac{.01595}{.0679} = .2349$$

and

$$\alpha_i = 1.54 \times .2349 = .362° \quad .$$

Continuing with data from Reference [1] we find from Table 16 that at the .7 radius station the variations of inflow angle from towing tank tests are

$$+\Delta\beta = 5.08°$$

and

$$-\Delta\beta = -5.69° \quad .$$

These values of $\Delta\beta$ provide for the distorted inflow caused by the ship's wake. These values can be summed with $\bar{\beta}$ in order to obtain the maximum and minimum values of attack angle for the two dimensional profile. As a result, in the following calculations we consider three attack angles: $\alpha|_{max} = 6.05°$, $\alpha_i = 0.362°$, and $\alpha|_{min} = -4.72°$.

Having determined C_{ℓ_i} as noted above, we can then determine the profile shape by adding the thickness distribution to the camber line. A plot of the resulting profile is shown in Figure 1. Once the profile ordinates have been defined, one can find the pressure distribution around the profile in accordance with potential theory. Three pressure distributions were obtained using a Douglas-Neumann program for the three values of α determined above. The resulting pressure distributions are plotted in Figure 2 below.

Once the pressure distributions are determined one can carry out boundary layer calculations. In the course of this study we considered a number of possibilities for executing such calculations. We have chosen Truckenbrodt's revised method [5] because of its simplicity. Basic steps in this calculation and the final results are given in Appendix A of this report. We need only mention here that such calculations were carried out for upper and lower surfaces of the profile of Figure 1 for the three attack angles called out in Figure 2 and for all six of the Reynolds numbers listed above. In all, thirty-six separate calculations were carried out. For those Reynolds numbers appropriate to model tests both laminar and turbulent boundary layers are expected to be present and a very simple criterion* was employed to fix the transition point. For the higher Reynolds numbers of the full scale propeller the boundary layer is predominantly turbulent. It should be noted that high precision is not the aim of these calculations. Our goal is to provide a straightforward engineering analysis which might lead to useful quality control requirements.

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*Appendix A

EFFECTS OF ROUGHNESS

Having dispensed with the necessary preliminaries, we are free to consider the matter of limits on roughness. The basic reference for all that follows is contained in the J.S.R. (1979) paper by Arndt, Holl, Bohn, and Bechtel [6] Figure 12 of this paper shows that an isolated two-dimensional triangular roughness element will produce a much stronger effect on cavitation onset than do isolated cones or triangular irregularities. Moreover, from the data of Figure 7 of the paper it appears that the triangular 2-D element produces a stronger effect than the circular-arc 2-D element. Therefore we are justified in considering the triangular element as the dominant case to consider because criteria based upon it will certainly contain other forms of roughness. We shall investigate the maximum roughness height which will permit no more than a relative effect, E, in increasing the onset of cavitation. In order to do this we can proceed as follows.

In this study we will neglect all cavitation scale effects and put the critical cavitation number for cavitation onset on a smooth body equal to the magnitude of $C_p\big|_{min}$. Thus

$$\sigma_{cs} = -C_p\big|_{min} \quad .$$

From the plot of Figure 2, page 25, below we see that at the three values of α we have

α	σ_{cs}	x/c
6.05°	2.77	0.01
.362°	0.35	0.45
-4.72°	2.83	0.01

The critical case is the third of these because the boundary layer will be thin and the critical cavitation number is large. However we will consider all three cases in order to compare the outcome at these three attack angles.

Next we can turn to the results summarized in Figure 7 of Arndt et al. [6]. For the triangular 2-D element we find

$$\sigma_{fp} = .152\left(\frac{h}{\delta}\right)^{.361}\left(\frac{U\delta}{\nu}\right)^{.196} ,$$

where σ_{fp} is the onset cavitation number for a protuberance on a flat plate having no pressure gradient, h is the height of the two-dimensional element, δ is the boundary layer thickness, U is the velocity at the edge of the boundary layer and ν is the kinematic viscosity. Suppose the flat plate has the same length as the profile chord c. Then the preceding equation can be expressed as

$$\sigma_{fp} = .152(Re)^{.196}\left(\frac{h}{c}\right)^{.361}\Big/\left(\frac{\delta}{c}\right)^{.165} .$$

Next we can introduce the superposition equation,

$$\sigma_{cR} = -C_{p_s} + (1 - C_{p_s})\,\sigma_{fp} ,$$

from Equation (6) of Arndt et.al [6]. Now let us define the relative effect of roughness by

$$E = \frac{\sigma_{cR} - \sigma_{cs}}{\sigma_{cs}} ,$$

where σ_{cR} is the onset cavitation number on the roughened body, σ_{cs} is the same quantity on the smooth body, and C_{p_s} is the pressure coefficient on the smooth body. Recalling our definition for E given above we can write

$$E = .152 \frac{(1-C_{p_s})(Re)^{.196}(h/c)^{.361}}{(-C_{p_s})(\delta/c)^{.165}} \quad .$$

Solving for h/c we get

$$\frac{h}{c} = \frac{E^{2.77}(\delta/C)^{.457}}{.0054(Re)^{.543}} \left(\frac{1 - C_{p_s}}{- C_{p_s}}\right)^{2.77} \quad . \qquad (A)$$

In the calculations to follow we will consider two values of E: 5 and 10%. For these two values we have $E^{2.77}/.0054 = .046$ and .313 respectively. The ratio of permissible h/c values in these cases is 6.8. The following table gives calculated values of h/c for these two values of E.

E	α°	x/c	-Re-					
			$3x10^5$	10^6	$3x10^6$	10^7	$3x10^7$	10^8
	6.05	0.01	$4.36x10^{-6}$	$1.72x10^{-6}$	$7.42x10^{-7}$	$2.91x10^{-7}$	$1.25x10^{-7}$	$4.94x10^{-8}$
5%	0.362	0.45	$1.69x10^{-5}$	$9.84x10^{-6}$	$6.53x10^{-6}$	$3.22x10^{-6}$	$1.61x10^{-6}$	$8.10x10^{-7}$
	-4.72	0.01	$7.01x10^{-6}$	$5.42x10^{-6}$	$5.69x10^{-7}$	$1.75x10^{-7}$	$7.48x10^{-8}$	$3.0x10^{-8}$
	6.05	0.01	$3.19x10^{-5}$	$1.17x10^{-5}$	$5.04x10^{-6}$	$1.98x10^{-6}$	$8.48x10^{-7}$	$3.36x10^{-7}$
10%	0.362	0.45	$1.15x10^{-4}$	$6.69x10^{-5}$	$4.44x10^{-5}$	$2.19x10^{-5}$	$1.09x10^{-5}$	$5.51x10^{-6}$
	-4.72	0.01	$4.81x10^{-5}$	$3.68x10^{-5}$	$3.87x10^{-6}$	$1.18x10^{-6}$	$5.08x10^{-7}$	$2.04x10^{-7}$
			model			full scale		

The values tabulated above must be supplemented by additional calculations for the minimum value of C_p on the lower surface at $\alpha = 0.362$, not because it is the minimum C_p at this value of α but because the value of x/c is 3.0% and although $C_p = -.159$ at this point the boundary layer will be very thin at this position.

E	-Re- 3x10^5	10^6	3x10^6	10^7	3x10^7	10^8
5%	1.33×10^{-5}	6.97×10^{-6}	2.99×10^{-6}	1.18×10^{-6}	1.11×10^{-6}	5.37×10^{-7}
10%	9.07×10^{-4}	4.74×10^{-5}	2.03×10^{-5}	8.02×10^{-6}	7.55×10^{-6}	3.65×10^{-6}
	model			full scale		

These new values can be compared with appropriate entries in the table above and we find that they are all larger than those associated with the minimum pressure coefficient. Therefore, it appears for the present three cases at least that all critical roughnesses are those calculated first.

On the otherhand it must be recognized that the data which underlie Equation (A) are obtained from roughness elements which were not smaller than 40 microns and for values of h/δ no smaller than .0136. This value of h/δ can be compared with the displacement and momentum thicknesses. For a laminar boundary layer on a flat plate we have

$$\frac{\delta_1}{\delta} = .344 \text{ and } \frac{\delta_2}{\delta} = .133 \quad .$$

Data in Table I of Reference [6] for triangular elements show height ratios as small as

$$\frac{h}{\delta} = .0136 \quad .$$

Since the experiments were carried out with turbulent boundary layers, it is of interest to compare this smallest height with the viscous sublayer height, δ_ℓ. We can estimate this quantity by using (see Schlichtig, op cit.)

$$\delta_\ell = \frac{5\nu}{v_*} \quad ,$$

where $v_* = \sqrt{\tau_o/\rho}$. Assuming a 1/7 power law profile and allowing for variations in pressure coefficient along a body, we would have for the shearing stress at the wall

$$\frac{\tau_o}{\rho} = U_o^{\,2}(1 - C_p)(.0296)(\mathrm{Re}\,\frac{x}{c})^{-1/5} \quad .$$

After some manipulation we get

$$\frac{\delta_\ell}{c} = 29\,\frac{(x/c)^{1/10}}{\mathrm{Re}^{9/10}\,\sqrt{1-C_p}} \quad .$$

If we put $C_p = 0$ for a flat plate and change 29 to 100, neglect $(x/c)^{1/10}$ and replace the $\mathrm{Re}^{9/10}$ with $\mathrm{Re} = U_o c/\nu$ we get the admissible roughness formula in Schlichting (page 660-661). It appears therefore that admissible roughness can be as large as four times the viscous sublayer height.

For the 1/7 power law profile we have

$$\frac{\delta}{c} = .37 \frac{(x/c)^{4/5}}{Re^{1/5}} \quad ,$$

and it follows that

$$\frac{\delta_\ell}{\delta} = \frac{78}{(x/c\ Re)^{7/10}} \quad ,$$

where we have retained the flat plate values. For a Reynolds number of 10^6 and $x/c = 1$ we get

$$\frac{\delta_\ell}{\delta} \doteq .005$$

This value is probably the largest of the viscous sublayer heights encountered by Arndt et al., but even so it is significantly smaller that their smallest h/δ noted above. If we were to multiply the ratio δ_ℓ/δ by 4 we find that h/δ is in the range of admissible roughness, as can be seen from data in Table 12.3 of Schlichting. Since the present analysis appears to show a possible effect on cavitation of roughness elements which are smaller than those of the experiments it would be of great interest to extend these experiments to even smaller roughness sizes. When one couples this finding with the fact in many situations encountered in the towing tank or water tunnel that laminar boundary layers are present, the extension of flat plate roughness data to include cavitation in laminar boundary layers would also be beneficial.

At this time it appears that our attempt, using 2-D triangular roughness, to define criteria for the onset of cavitation on marine propellers is hampered by a lack of data although the methods by which such rational standards might be developed seem straightforward.

As we have noted, the preceding calculations assume an isolated two-dimensional roughness of triangular shape because of its strong effect on cavitation inception. On the other hand if we were to go to the other extreme and consider distributed roughness only, we may be justified in using the usual criterion based on skin friction which defines admissible roughness. We base this assertion upon the correlation given in Figure 6 of Reference [6]. In this case we would have

$$\frac{h}{c} \sim \frac{100}{Re\sqrt{1-C_p(x/c)}} \qquad \text{(B)}$$

for the admissible height of distributed roughness. In this case there would be no measurable effect on cavitation inception; that is, in terms of the above terminology $E = 0$. Clearly, the critical value of h would be found at $C_p\big|_{min}$.

EFFECTS OF A CONTOUR IRREGULARITY

The formulation of a method by which one can examine the effects of variations in profile contour seem somewhat less demanding than the preceding considerations. One way is to use potential theory to calculate the pressure distribution of the deformed profile from which the cavitation performance can be reassessed. The chief problem with this approach is

that not all organizations have a computer program possessing the "numerical flexibility" which would enable them to handle small bumps or waves without difficulty. For those who do not have such a "flexible" program, we would propose the following approximate procedure.

The basic idea here is to use the superposition equation from Reference [6] as quoted above. For σ_{fp} one would simply use the magnitude of $C_p\big|_{min}$ of one of the bumps given in Appendix B of this report. See Figure 3 below. The conditions under which this procedure is valid are shown on Figure 2 of Reference [6]. The curves and the data given there show that if

$$h/\delta \geq 4 \quad ,$$

then the proposed procedure should work satisfactorily. Here h is the maximum height of the bump. Otherwise the methods which apply to smaller roughness elements as discussed above should be used.

In terms of the bumps and pressure distributions of Appendix B we may note that the parameter ε is given by

$$\varepsilon = h/\ell \quad ,$$

where ℓ is the length of the bump (taken as unity in those calculations) and h is the actual height of the bump. Therefore we can write the <u>criterion of applicability</u> as

$$\frac{\ell}{\delta}\,\varepsilon \geq 4 \quad .$$

In those cases where the observed bump is not well represented by either of those given in Appendix B one can develop the measured contour deviation about

a straight line. Then he can superpose bumps having various values of ε along this line until the observed deviation from the line is fairly well represented. After that he can superpose the pressure distributions from all bumps. Values of x (or x/ℓ), η/ε and C_p/ε tabulated in Appendix B could be useful in this phase of the calculations. The σ_{fp} will be equal to the magnitude of $C_p\big|_{min}$ obtained from the composite pressure distribution.

As a simple illustration, let us suppose that only one bump is needed to characterize at least approximately a typical deformation. If we settle on Case #2 from Appendix B we see that

$$C_p\Big|_{min} = -.68\varepsilon \quad .$$

Then since

$$E = \left(\frac{1-C_{p_s}}{-C_{p_s}}\right)\sigma_{fp} \quad ,$$

we have

$$E = .68\left(\frac{1-C_{p_s}}{-C_{p_s}}\right)\varepsilon \quad .$$

Now if we put E = .1 we have

$$\varepsilon = .147\left(\frac{-C_{p_s}}{1-C_{p_s}}\right)$$

Finally we can use the "criterion of applicability" from above and write

$$\frac{\delta}{\ell} \leq .0368 \left(\frac{-C_{p_s}}{1-C_{p_s}}\right) \quad .$$

and we also see that it leads to the alternate expression

$$\varepsilon \leq .147 \left(\frac{-C_{p_s}}{1-C_{p_s}}\right) \quad .$$

As an application of these results suppose that we consider the critical case from the plot of Figure 2 which is found at $\alpha = -4.72$. $C_p\big|_{min}$ for this case is -2.84 and we find

$$\varepsilon \leq .109$$

$$\frac{\delta}{\ell} \leq .0272$$

On the other hand we find from Appendix A that

$$\frac{\delta}{c} = .00015 \text{ when } Re = 10^6 \quad .$$

Hence it follows that

$$\frac{\ell}{c} \geq .005 \quad .$$

These data, together with the fact that the bump is given by

$$\eta(x) = \varepsilon(1-4x^2)^2 \quad ,$$

where x is measured from the center of the bump and is an actual distance normalized by ℓ and η is the normalized ordinate of the bump; serves to characterize a permissible deformation which will produce a relative error in cavitation number of about 10%. The effect of more complex bumps can be explored by use of the composite pressure distribution discussed above.

CONCLUSIONS

An investigation which makes use of the most complete information available on the effects of surface asperities with respect to cavitation inception, has sought to determine how these data might be used to formulate quality control criteria for cavitation inception on model propeller blades. The OBO propeller was used as an example. It was found that the dominant effect is caused by the two-dimensional isolated triangular element. In this case the data are not complete enough to permit the formulation of a roughness criterion. On the other hand, if one considers the effects of distributed roughness or single bumps which protrude well outside the boundary layer one can formulate methods for obtaining simple quality control criteria for these two cases.

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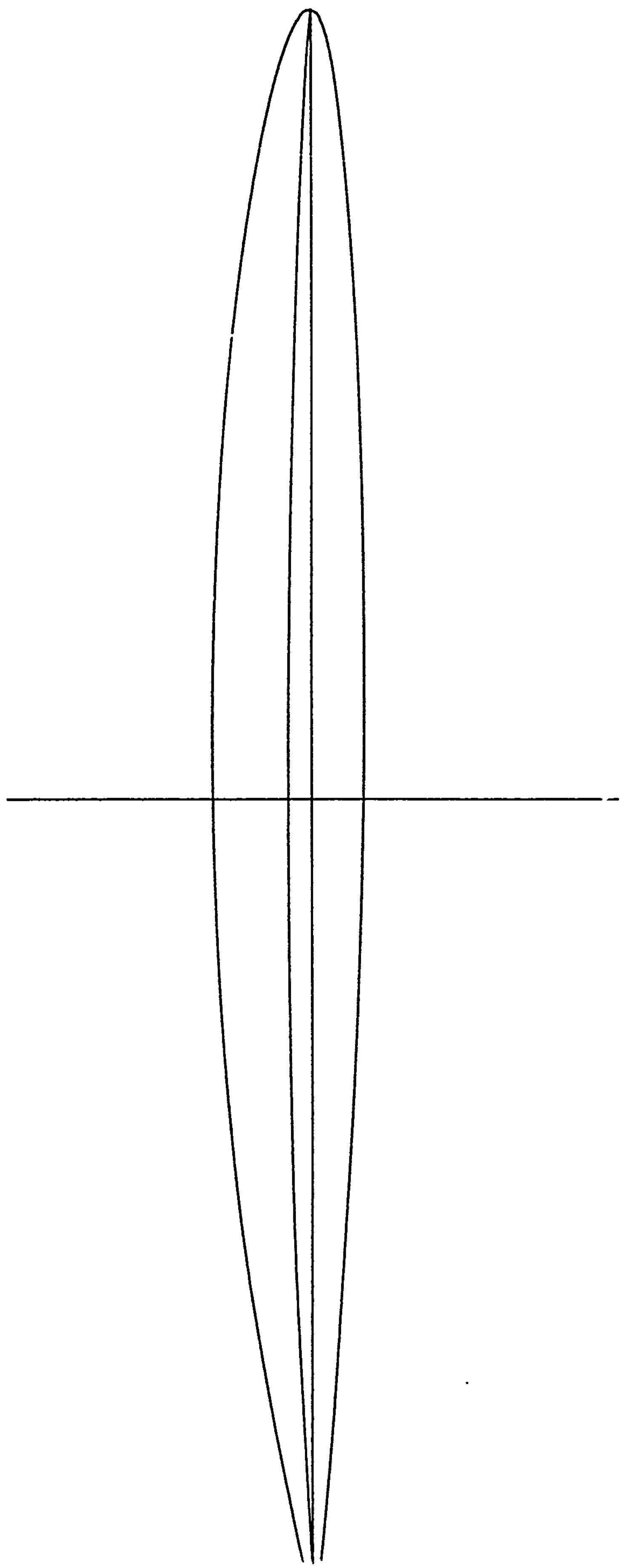

Figure 1. Modified NACA66 Blade Section

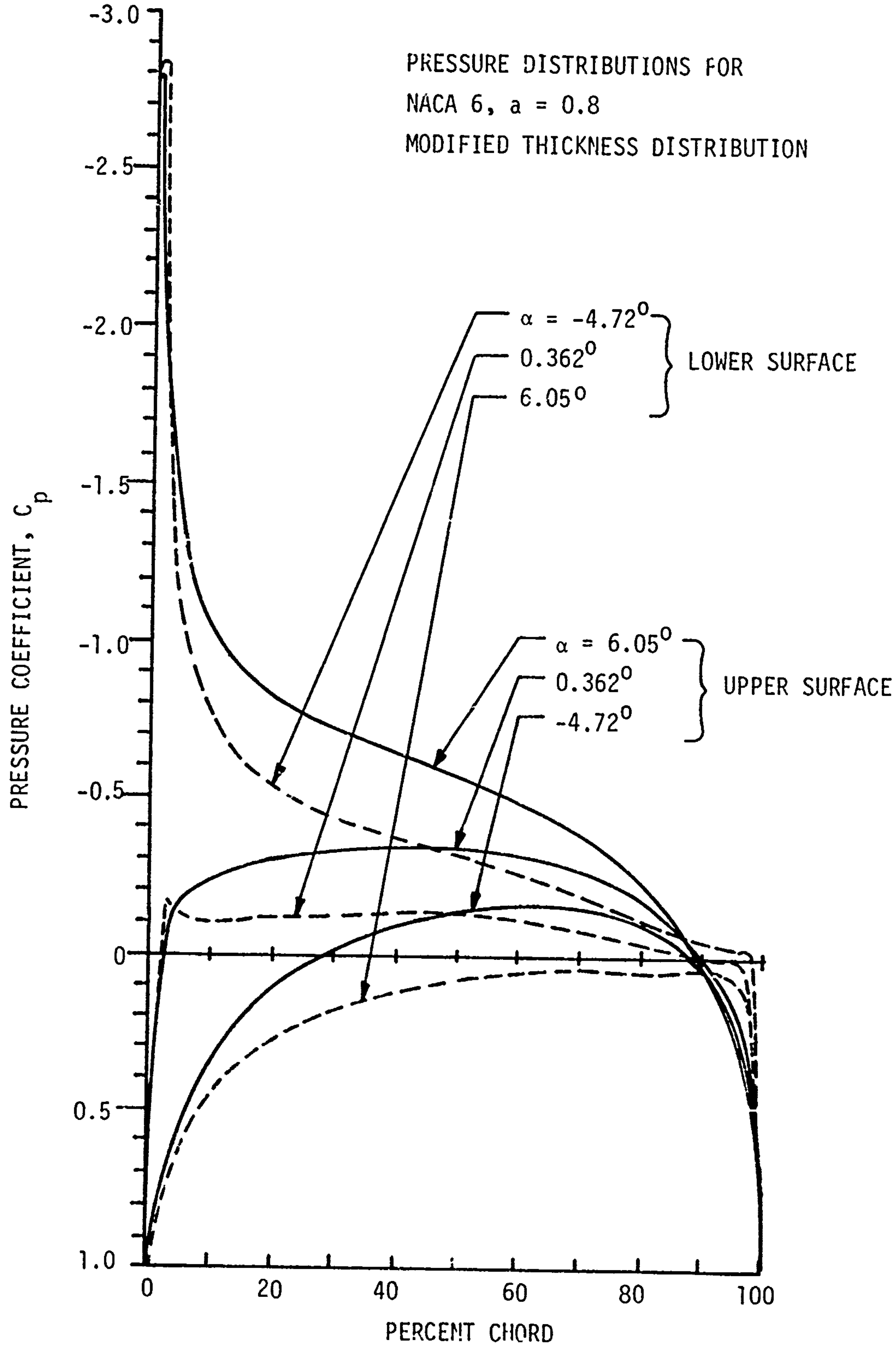

Figure 2. Calculated Pressure Distribution on Modified NACA6 (a = .8) Profile of Figure 1

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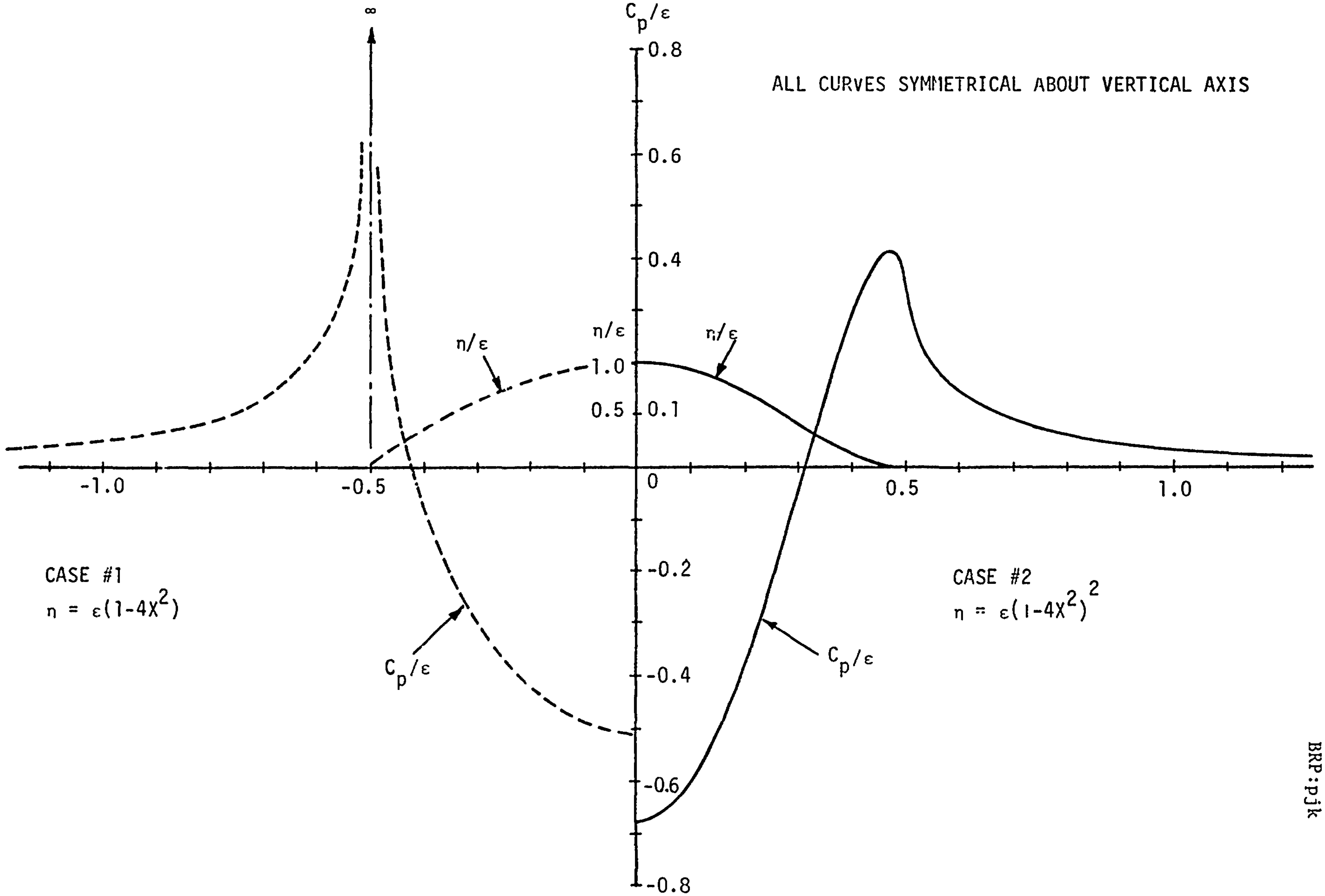

Figure 3. Pressure Distributions from Appendix B of Two Bumps on a Flat Plate. Case #1: Linearized "Circular" Bump (Parabolic Contour). Case #2: Bump with Cusp.

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APPENDICIES

A: Boundary Layer Thickness

Truckenbrodt's Revised Integral Method.

See: Boundary-Layer Theory by H. Schlichting, 7th Edition, McGraw-Hill Book Company, New York, 1980, Chapter XXII.

Basic Equations Used: Let x be arc length. Then

$$R_3(x) = \frac{\delta_3 U(x)}{\nu} = \frac{\delta_3}{c}\frac{c\,U_o}{\nu} \cdot \frac{U(x)}{U_o} = \left(\frac{\delta_3}{c}\right) \mathrm{Re}\sqrt{1-C_p(x)} \quad ,$$

where

$$\delta_3(x) = \text{energy thickness} = \int_o^{\delta} [1-(u/U(x))^2]\ (u/U)dy \quad ,$$

ν = kinematic viscosity,

c = profile chord length,

U(x) = velocity at edge of boundary layer,

u(y) = velocity profile at any arc length, x,

U_o = free stream velocity

Re = Reynolds number based on free stream velocity and chord length

$\delta(x)$ = boundary layer thickness at the arc length x

Laminar Boundary Layer:

If we assume that u(y) can be approximated by the Blasius profile we get

$$\frac{\delta}{c} = 4.788\,\frac{\delta_3}{c} \quad ,$$

as can be found from a simple Simpson's rule integration as shown below.

Calculations for the ratio δ/δ_s for the Laminar Boundary Layer.

Blasius Flat Plate Solution (see Schlichting p. 139, 7th Edition).

$$\delta_3 = \frac{\delta}{5} \int_0 \overbrace{f' (1-[f']^2)}^{F(\xi)} d\xi = \frac{\delta}{5} I$$

	ξ	F		ξ	F
1.	0	0	22.	4.2	.06284
2.	.2	.06612	23.	4.4	.04653
3.	.4	.13043	24.	4.6	.03373
4.	.6	.19107	25.	4.8	.02397
5.	.8	.24616	26.	5.0	.01669
6.	1.0	.29392	27.	5.2	.01140
7.	1.2	.33272	28.	5.4	.00764
8.	1.4	.36128	29.	5.6	.00502
9.	1.6	.37876	30.	5.8	.00323
10.	1.8	.38489	31.	6.0	.00204
11.	2.0	.38000	32.	6.2	.00126
12.	2.2	.36505	33.	6.4	.00078
13.	2.4	.34158	34.	6.6	.00046
14.	2.6	.31154	35.	6.8	.00026
15.	2.8	.27708	36.	7.0	.00016
16.	3.0	.24045	37.	7.2	.00008
17.	3.2	.20366	38.	7.4	.00004
18.	3.4	.16846	39.	7.6	.00002
19.	3.6	.13616	40.	7.8	0
20.	3.8	.10756	41.	8.0	0
21.	4.0	.08311			

$n = 40$

$h = 0.2$

Using a TI 59 with ML 10 for Simpson's Rule we get $I = 1.04434$. Therefore

$$\delta = \frac{5\delta_3}{1.04434} = 4.78771\delta_3 \quad .$$

The calculator results are shown on the next page.

ML 10 - SIMPSON'S RULE

40. INTERVALS
0.2 h

1.04433867 ← I

List Data registers.

0.	00
6.	01
0.	02
0.2	03
1.04433867	04
40.	05
0.	06
0.06612	07
0.13043	08
0.19107	09
0.24616	10
0.29392	11
0.33272	12
0.36128	13
0.37876	14
0.38489	15
0.38	16
0.36505	17
0.34158	18
0.31154	19
0.27708	20
0.24045	21
0.20366	22
0.16846	23
0.13616	24
0.10756	25
0.08311	26
0.06284	27
0.04653	28
0.03373	29
0.02037	30
0.01669	31
0.0114	32
0.00764	33
0.00502	34
0.00323	35
0.00204	36
0.00126	37
0.00078	38
0.00046	39
0.00026	40
0.00016	41
0.00008	42
0.00004	43
0.00002	44
0.	45
[illegible]	46

Calculator results for the determination of δ/δ_3

Then, following Schlichting, page 682, we can put $\nu_\ell' = .917\nu$ and

$$\frac{\delta}{c} = \frac{4.788}{\sqrt{Re}} \left[\frac{\int_o^{\xi_1} [1-C_p(\xi)]^{2.5} d\xi}{.917\ [1-C_p(\xi_1)]^3} \right]^{1/2} = \frac{5}{\sqrt{Re}} \left[\frac{\int_o^{\xi_1} [1-C_p(\xi)]^{2.5} d\xi}{[1-C_p(\xi_1)]^3} \right]^{1/2} . \qquad \text{(A-1)}$$

It is worth observing that when $C_p = 0$ in the laminar boundary Equation (A-1) that we get

$$\frac{\delta}{c} = \frac{5}{\sqrt{Re}} \left(\frac{x}{c}\right)^{1/2}$$

which is the well known Blasius result. Moreover in Equation (A-1) the quantity ξ is given by

$$\xi = x/c \quad ,$$

and as is customary, at the forward stagnation point one puts $\delta = 0$.

Equation (A-1) is used to calculate the laminar boundary layer thickness from the nose of the profile to the transition point. For many purposes it is permissible to use a simple criterion in order to estimate the location of transition. One can suppose that transition may occur at an arc-length Reynolds number of

$$Re_t = 3 \times 10^5$$

or in the neighborhood of $C_{p_{min}}$, whichever occurs first. The criterion is probably most appropriate if the outer flow is highly turbulent as in the wake of a ship although its chief virtue is simplicity rather than accuracy.

Once the transition point x_1 has been selected one can calculate the laminar energy-layer Reynolds number,

$$R_3(x_1) = \frac{\delta(x_1)}{c} \frac{Re\sqrt{1-C_p(x_1)}}{4.788} \quad , \tag{A-2}$$

for the transition point only. One assumes that the energy layer δ_3 is continuous at transition.

Turbulent Boundary Layer

Once the laminar boundary layer thickness distribution and the location of transition have been determined, calculations for the turbulent boundary layer can be made. The formula for the energy thickness Reynolds number is

$$R_3(x) = \left\{ [R_3(x_1)]^{1.152} + \frac{Re}{\nu'} \frac{\int_{\xi_1}^{\xi} [1-C_p(\xi)]^{1.65} d\xi}{[1-C_p(x)]^{1.15}} \right\}^{\frac{1}{1.152}} \quad , \tag{A-3}$$

where $R_3(x_1)$ is given by Equation (A-2), $\xi = x/c$ as before and $\xi_1 = x_1/c$. Schlichting suggests that for ν' one should use $\nu' = 80\nu$. The boundary layer thickness can be determined from the energy thickness if it is assumed that the velocity profile can be approximated by a one-seventh power law profile which is known to give rather good results for flat-plate skin friction at Reynolds numbers less than 10^7. As a result one finds that

$$\delta = 40\ \delta_3(x)/7 \quad ,$$

from which it follows that

$$\frac{\delta(x)}{c} = \frac{5.714\ R_3(x)}{Re\sqrt{1-C_p(x)}} \quad . \tag{A-4}$$

In the course of preliminary calculations based upon Equations (A-1) through (A-4) the recommended value of $\nu' = 80\ \nu$ was used. Surprisingly, when laminar and turbulent boundary layer calculations were started from the nose of a profile it was found that the laminar boundary layer thickness was larger than the turbulent boundary layer thickness for a considerable range of arc lengths starting from the nose.

Therefore we conducted a short investigation to see if a better choice for the value of ν' could be made. We considered a flat plate of length c at $Re = 10^6$. The laminar boundary layer thickness for this case can be found from the formula at the top of page 30. The corresponding turbulent boundary layer thickness for a one-seventh power low profile is known to be

$$\frac{\delta(x)}{c} = \frac{.37(x/c)^{4/5}}{Re^{1/5}} \quad ,$$

and if one puts $C_p(\xi) = R_3(x_1) = 0$ in Equations (A-3) and (A-4) it follows that

$$\frac{\delta(x)}{c} = \frac{5.714\left(\frac{\nu}{\nu'}\ \frac{x}{c}\right)^{.868}}{(Re)^{.132}} \quad .$$

If one equates these two values of turbulent boundary layer thickness he finds that the ratio ν/ν' is

$$\frac{\nu}{\nu'} = \frac{.0427}{[(x/c)Re]^{.0452}} \quad . \tag{A-5}$$

Equation (A-5) can be used to explore various possible values for ν'. For example we have used $Re = 10^6$ as noted above and we considered a few values of x/c as tabulated below.

$\underline{Re = 10^6}$

$\frac{x}{c}$	$\frac{\nu'}{\nu}$
.5	65
.1	58
.05	38

Table showing the effect upon possible values of ν' for three transition point locations on a flat plate.

Next, two likely values of ν' were used to compare 1/7 power and Truckenbrodt boundary layer thicknesses directly at $Re = 10^6$ and $x/c = .08$ in which case $\delta/c\big|_{1/7} = .00309$.

$$\text{For } \nu' = 50\nu \quad , \quad \frac{\delta}{c}\bigg|_T = .00345$$

$$\text{For } \nu' = 55\nu \quad , \quad \frac{\delta}{c}\bigg|_T = .0032$$

As a result of these comparisons it seemed best to take

$$\nu' = 55\nu$$

for the present calculations. Nevertheless as a final illustration of the effect of changing from $\nu' = 55\nu$ to $\nu' = 80\nu$ on a typical profile is illustrated by the data on page 39 below, which are given for $Re = 3 \times 10^5$

and Re = 10^8 when $\nu' = 80\nu$. These data can be compared with the corresponding results for $\nu' = 55\nu$ on pages 36 and 38 respectively.

Numerical Results and Calculator Program

Equations (A-1) through (A-4) were coded for execution on a TI-59 programmable calculator. For the record, we give a description of the procedure and other details of this phase of the calculations.

Data Registers

R_{00} - for DSZ index

R_{01} - used, $R_{\overrightarrow{02}}$j and R_3 laminar

R_{03} - h in Simpson's rule

R_{04} - I, Simpson's rule result

R_{05} - j_{max} = total no of C_p entries

$\left.\begin{array}{l} R_{06} - f_0,\ k = 6 \\ R_{07} - f_1,\ k = 7 \\ R_{08} - f_2,\ k = 8 \end{array}\right\}$ in Simpson's rule

R_{09} - $j(x_1)$ = an odd integer denoting location of x_1 (transition)

$R_{10} = C_p(0) - R_{60} = C_p(x_{max})$, a total of 51 $C_p(x)$ entries can be used

$\therefore 1 \le j \le 51$

R_{61} - blank

R_{62} - $[1-C_{p_j}], 1 \le j \le 51$

R_{63} - j

R_{64} - k

R_{65} - Re

R_{66} - ΣI from R_{04}

R_{67} - δ/c laminar

R_{68} - $R_3(x)$ turbulent

R_{69} - f_k, $6 \le k \le 8$ where $f_k = [1-C_{p_k}]^{1.65}$

Preliminaries

(a) Partition memory $\boxed{7}$ $\boxed{\text{2nd}}$ $\boxed{\text{Op}}$ $\boxed{17}$, display 399.69. To check, $\boxed{\text{2nd}}$ $\boxed{\text{Op}}$ $\boxed{16}$.

(b) Enter program from magnetic card $\boxed{0}$ $\boxed{\text{INV}}$ $\boxed{\text{2nd}}$ $\boxed{\text{WRITE}}$ for both 1 and 2.

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Procedure

(a) enter data: $\boxed{h}$ $\boxed{ST\emptyset}$ $\boxed{03}$ $\boxed{Jmax}$ $\boxed{ST\emptyset}$ $\boxed{05}$ $\boxed{j(x_1)}$ $\boxed{ST\emptyset}$ $\boxed{09}$

$\boxed{RE}$ $\boxed{ST\emptyset}$ $\boxed{65}$

(b) enter Cp_j: $\boxed{2nd}$ $\boxed{Pgm}$ $\boxed{10}$ $\boxed{0}$ $\boxed{C}$ $\boxed{R/S}$ $\boxed{0}$ $\boxed{R/S}$ $\boxed{0}$ $\boxed{R/S}$ $\boxed{0}$

$\boxed{R/S}$ ← this moves pointer to R_{10}. $\boxed{Cp_1}$ $\boxed{R/S}$ ...

$\boxed{Cp_{jmax}}$ $\boxed{R/S}$

(c) Run program: $\boxed{CLR}$ $\boxed{RST}$ $\boxed{R/S}$

—//—

Program Listing and Data Register contents given on pages 55 - 57. Note value of ν' in Program Memory addresses 306 and 307.

Specific boundary layer calculation results for OBO propeller are given on pages 36 - 54.

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3000000. Re

0.04 x/c
.0009376418 δ/c

0.08
.0019023488

0.12
.0024701081

0.16
.0029294425

0.2 laminar boundary layer
.0033101646

0.24
.0036683173

0.28
0.004005852

0.32
.0043112699

0.36
.0046029608

0.4
.0048822734

0.44
.0051374603

0.48 x/c
.0054083593 δ/c
394.0277918 R_θ
.0064542164 δ/c
transition

0.52
.007[illegible]

0.56
.0093435161

0.6 turbulent boundary layer
.0107953263

0.64
.[illegible]

0.68
.[illegible]

0.72
.0153751967

0.76
.0170827223

0.8
0.01900231

0.84
.0215967958

0.88
.0243971632

0.92
.0292914488

0.96
.0360208913

1000000.

0.04
.0005135673

0.08
.0010419593

0.12
.0013529329

0.16
.0016045217

0.2
.0018130518

0.24
.0020092201

0.28
.002[illegible]40355

0.32
.0023613798
569.9173807
0.00281[illegible]

0.36
.0034[illegible]

0.4
.0053[illegible]

0.44
.0065919314

0.48
.0077832053

0.52
.0089988211

0.56
0.010215500

0.6
.0114606336

0.64
.0127500398

0.68
.0141143953

0.72
0.015574235

0.76
0.0171578[illegible]

0.8
.0189817176

0.84
.0215653229

0.88
.0249[illegible]

0.92
[illegible]

0.96
[illegible]

Upper Surface
Design $\alpha_i = 0.362°$

3000000.

0.04
.0002965012

0.08
.0006015755
413.6391705
.0007179203

0.12
.0011845458

0.16
.0031015664

0.2
.0041020305

0.24
.0051403602

0.28
.0061329768

0.32
.0070921016

0.36
.0080333762

0.4
.0089842969

0.44
.0098943447

0.48
.0108467404

0.52
0.011870836

0.56
0.012913222

0.6
.0140066658

0.64
.0151653424

0.72
.0177997613

0.76
.0193267107

0.8
0.021120936

0.84
0.0[illegible]9268[illegible]9

0.88
.0274130114

0.92
.0324843673

0.96
.0405975963

10000000.

0.04
.0001624042
363.5666098
.0001938133

0.08
.0012484488

0.12
.0022990653

0.16
.0031877932

0.2
.0040229164

0.24
0048550071

0.28
.0056524513

0.32
.0064841[illegible]7

0.36
.0072753417

0.4
.0080755409

0.44
.0088425472

0.48
.0096489663

0.52
.0105221947

0.56
.0114124465

0.6
.0123489776

0.64
.0133440408

0.68
.0144272451

0.72
.0156160003

0.76
.0159848275

0.8
.0135025544

0.84
.020[illegible]57[illegible]199

0.88
.0239962079

0.92
.0264490041

0.96
.0355604[illegible]6

Upper Surface
Design $\alpha_i = 0.342°$

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30000000.

0.04
.0004724448

0.08
.0013375542

0.12
.0021766187

0.16
.0029285496

0.2
.0036380299

0.24
.0043494203

0.28
.0050593536

0.32
.0057478721

0.36
.0064313292

0.4
.0071174302

0.44
.0077778881

0.48
.0084737386

0.52
.0092289953

0.56
.0099994463

0.6
.0108107699

0.64
0.011673617

0.68
.0126139685

0.72
0.013646715

0.76
.0147935809

0.8
0.016158505

0.84
.0182126616

0.88
.0209524713

0.92
.0248369054

0.96
.0310581546

100000000.

0.04
.0004030512

0.08
.0011837473

0.12
.0018569124

0.16
.0024983981

0.2
.0031036684

0.24
.0037105683

0.28
.0043182251

0.32
.0049036126

0.36
0.005486682

0.4
.0060720071

0.44
.0066354555

0.48
.0072290981

0.52
.0078734211

0.56
.0085307066

0.6
.0092228613

0.64
.0099559716

0.68
.0107612023

0.72
.0116422569

0.76
.0126206687

0.8
.0137851099

0.84
.0155375476

0.88
.0178749283

0.92
.0211883086

0.96
.0264962676

Upper Surface
Design $\alpha_i = 0.362°$

300000.

0.04
.0009376413

0.08
.0019023488

0.12
.0024701081

0.16
.0029294425

0.2
.0033101646

0.24
.0036683173

0.28
0.004005852

0.32
.0043112699

0.36
.0046009608

0.4
.0048822734

0.44
.0051374603

0.48
.0054082563
394.0277938
.0064542164

0.52
.0074635519

0.56
.0084683804

0.6
.0094854955

0.64
.0105271771

$\nu' = 80\nu$, DESIGN $\alpha_i = 0.362°$ Upper Surface.

0.68
.0116146284

0.72
.0127634949

0.76
.0139946322

0.8
0.015385856

0.84
.0172822115

0.88
.0197017415

0.92
.0230016833

0.96
.0280911424

100000000.

0.04
.0002911413

0.08
0.000855072

0.12
.0013413283

0.16
.0018047012

0.2
.0022419142

0.24
.0026803043

0.28
.0031177964

0.32
.0035420918

0.4
.0043860738

0.44
0.004793077

0.48
.0052218908

0.52
.0056873132

0.56
0.006162099

0.6
.0066620723

0.64
.0071937967

0.68
.0077732827

0.72
.0084097066

0.76
.0091164558

0.8
.0099575821

0.84
.0112234439

0.88
.0129118352

0.92
.0153055945

0.96
.0191394021

$\nu' = 80\nu$ Design $\alpha_i = 0.362°$ Upper Surface.

Illustrating the effect of changing $\nu' = 55\nu$

300000.

0.04
.0012546472
84.6460003
.0014972961

0.08
0.003280681

0.12
.0048554775

0.16
.0063365071

0.2
.0077284185

0.24
.0091087414

0.28
.0104725978

0.32
.0117913058

0.36
.0130887217

0.4
.0143804516

0.44
.0156147706

0.48
0.016901508

0.52
.0182890569

0.56
.0196864444

0.6
0.021140644

0.64
.0226556808

0.68
.0242723047

0.72
.0260029472

0.7[illegible]
0.0278095[illegible]

0.[illegible]
0.029829[illegible]9

0.84
.0314327358

0.88
.0328029596

0.92
.0346754874

0.96
.0370996329

1000000.

0.04
.0006871986
154.5417459
.0008201029

0.08
.0023911334

0.12
.0037602971

0.16
.0050404595

0.2
.0062405811

0.24
.0074377516

0.28
.0085989369

0.32
.0097305293

0.36

0.4
.0119499067

0.44
.0130074652

0.48
.0141089661

0.52
.0152955959

0.56
.0164902529

0.6
.0177329005

0.64
.0190270274

0.68
.0204073319

0.72
.0218844464

0.76
0.023426065

0.8
.0251492058

0.84
.0265175783

0.88
.0276877322

0.92
.0292852685

0.96
.031352[illegible]

Lower Surface
Design $\alpha_i = 0.362^\circ$

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3000000.

0.04
.0003967543
267.6741558
.0004734866

0.08
.0018673537

0.12
.0030659324

0.16
.0041821153

0.2
.0052267881

0.24
.0062586347

0.28
.0072756406

0.32
.0082578309

0.36
.0092230411

0.4
0.01018297

0.44
.0111000692

0.48
.0120547969

0.52
.0130827365

0.56
.0141174371

0.6
.0151934176

0.64
0.016313723

0.72
.0187864753

0.76
.0201202685

0.8
.0216107965

0.84
.0227948835

0.88
.0238077233

0.92
.0251897227

0.96
.0269772887

10000000.

0.04
.0002173113
486.7039106
.0002593393

0.08
.0014758664

0.12
.0025072945

0.16
.0034648479

0.2
.0043599536

0.24
.0052431286

0.28
.0061130317

0.32
.0069528908

0.36
0.007777982

0.4
.0085983302

0.44
.0093820344

0.48
0.010197616

0.52
.0110754067

0.56
.0119588578

0.6
0.012877389

0.64
.0138336139

0.68
.0148530959

0.72
.0159437009

0.76
.0170817043

0.8
.0183532519

0.84
.0193636298

0.88
.0202280417

0.92
.0214070732

0.96
.0229317861

Lower Surface
Design $\alpha_i = 0.0362°$

30000000.

0.04
0.000702417

0.08
.0017112778

0.12
.0025723721

0.16
.0033792262

0.2
.0041364489

0.24
.0048884047

0.28
.0056320345

0.32
.0063503765

0.36
.0070571528

0.4
.0077611964

0.44
0.008433027

0.48
.0091347991

0.52
.0098935034

0.56
.0106577044

0.6
.0114536817

0.64
.0122835724

0.68
.0131700605

0.72
0.014119868

0.76
.0151117488

0.8
0.016222192

0.84
.0171004331

0.88
.0178490865

0.92
.0186766001

0.96
.0202100413

100000000.

0.04
.0005992445

0.08
.0014599217

0.12
.0021945367

0.16
.0028833902

0.2
.0035288786

0.24
.0041703855

0.28
.0048047894

0.32
.0054176199

0.36
.0060205873

0.4
0.006631207

0.44
.0071943663

0.48
.0077930607

0.52
.0084403249

0.56
.0090922704

0.6
.0097713409

0.64
.0104793355

0.68
.0112355867

0.72
.0120459121

0.76
0128921004

0.8
.0138594424

0.84
.0145886856

0.88
.0152273752

0.92
.0161029655

0.96
0.017241548

Lower Surface
Design $\alpha_i = 0.362$

300000.

0.04
.0015608912
149.7837723
.0018627678

0.08
0.003732662

0.12
.0055490341

0.16
0.007275482

0.2
.0089630575

0.24
.0106526293

0.28
.0122929082

0.32
.0139136793

0.36
.0155342076

0.4
.0171270695

0.44
.0187232699

0.48
.0204788167

0.52
0.022301576

0.56
0.0242037[illegible]96

0.6
.0262603733

0.64
.0284823377

0.68
.0309664627

0.72
.0337351769

0.76
.0369307918

0.8
.0413830652

0.84
.0478748142

0.88
0.056648394

0.92
.0710190852

0.96
.0769179697

1000000.

0.04
.0008549353
273.4665028
.0010202799

0.08
.0026435913

0.12
0.004207965

0.16
0.005691551

0.2
.00713825[illegible]1

0.24
.00278642[illegible]8

0.28
.00558[illegible]

0.32
.0113[illegible]76538

0.36
0.012763528

0.4
.0141255594

0.44
.0154699326

0.48
.0169888489

0.52
.0185445153

0.56
.0201725053

0.6
.0219211618

0.64
.0238153107

0.68
.0259320936

0.72
.0281907468

0.76
.0310123659

0.8
.0345025492

0.84
.0403275[illegible]3

0.88
.0477958[illegible]8

0.92
.0[illegible]

0.96
.05[illegible]

Upper surface
$\alpha = 6.05^{\circ}$

3000000.

0.04
0004935971
473.6578769
.0005890589

0.08
.0020190109

0.12
.0033819654

0.16
.0046716732

0.2
.0059284952

0.24
.0071835703

0.28
.0084010807

0.32
.0096030363

0.36
.0108037606

0.4
.0119835495

0.44
.0131651701

0.48
.0144625057

0.52
.0158086272

0.56
.0172170208

0.6
.0187294401

0.64
.0203673108

[illegible]

0.72
.0242360799

0.76
.0265882463

0.8
0.029863088

0.84
.0346365544

0.88
.0410886754

0.92
.0516593351

0.96
.0559996618

10000000.

0.04
.0002703543
864.7770125
.0003226408

0.08
.0015658644

0.12
.0027353681

0.16
.0038397064

0.2
.0049147341

0.24
0.005987436

0.28
.0070277529

0.32
0.008054507

0.36
.0090799597

0.4
.0100874214

0.44
.0110962962

0.48
.0122034976

0.52
.0133521385

0.56
.0145537331

0.6
.0158438556

0.64
.0172407859

0.68
.0188013351

0.72
.0205397464

0.76
.0225451698

0.8
.0253367686

0.84
.0294055319

0.88
.0349052054

0.92
.0439161426

0.96
.0476162741

Upper Surface
$\alpha = 6.05^{\circ}$

30000000.

0.04
.0001560891
1497.837723
.0001862768

0.08
.0012779455

0.12
.0022933292

0.16
.0032507615

0.2
.0041821632

0.24
.0051111183

0.28
.0060118789

0.32
.0069007563

0.36
.0077883808

0.4
0.008660375

0.44
.0095335167

0.48
.0104915258

0.52
.0114853936

0.56
.0125247855

0.6
.0136407529

0.64
.0148490099

0.68
.0161986702

0.72
.0177020666

0.76
.0194362776

0.8
.0218501064

0.84
.0253681122

0.88
.0301234795

0.92
.0379149576

0.96
.0411145013

100000000.

0.04
.0000854935
2734.665028
0.000102028

0.08
.0010458544

0.12
.0019143769

0.16
.0027324596

0.2
.0035279102

0.24
.0043210123

0.28
.0050299522

0.32
.0058486643

0.36
.0066062326

0.4
.0073504264

0.44
.0080955552

0.48
.0089129753

0.52
.0097608504

0.56
.0106476846

0.6
.0115997001

0.64
.0126303885

0.68
.0137816316

0.72
.0150639576

0.76
.0165430995

0.8
.0186017645

0.84
.0216020505

0.88
0.025657646

0.92
.0320027056

0.96
.0350315532

Upper Surface
$\alpha = 6.05°$

300000.

0.04
.0010477876

0.08
.0017955816

0.12
.0022666061

0.16
.0026793723

0.2
.0030243667

0.24
.0033451912

0.28
.0036515341

0.32
.0039243218

0.36
.0041827611

0.4
.0044260237

0.44
.0046472245

0.48
0.004867167

0.52
.0051148566

0.56
.0053547413

0.6
.0055996502

0.64
.0058455203

Lower Surface

0.32
.0037119391

0.36
.0049118113

0.4
.0060310817

0.44
.0070826136

0.48
.0081041583

0.52
.0091492846

0.56
.0101763168

0.6
.0112106699

0.64
.0122493383

0.68
.0133295067

0.72
.0144464778

0.76
.0155821534

0.8
.0168163146

0.84
.0178374157

0.88
.0186429144

0.92
.0196953464

0.96
.0213090116

1.
.0251800968

0.68
.0061098284

0.72
.0063874935

0.76
.0066682963

0.8
0.006987488

0.84
.0072071126

0.88
.0073330886
450.0794852
.0087513092

0.92
0.010162042

0.96
0.01169864

1.
.0172139093

1000000.

0.04
.0005738969

0.08
.0009834806

0.12
0.001240324

0.16
.0014675526

0.2
.0016565118

0.24
.0018333367

0.28
.0030000276
372.904255
.0023958372

5000000.

0.04
.0003313365

0.08
.0005678128
244.1671758
.0006776278

0.12
.00[illegible]231729

0.16
.0027480266

0.2
.0035894509

0.24
.00440905[illegible]08

0.28
.0052222702

0.32
0.006005714

0.36
.0067816882

0.4
.0075469519

0.44
.0082670265

0.48
.00903542[illegible]7

0.52
.00[illegible]8470644

0.56
.010[illegible]583167

0.6
.01149344[illegible]6

0.64
.0123449581

0.68
.0132490079

0.72
.0141979837

0.76
0.015170189

0.8
.0162462101

0.84
.017[illegible]076654

0.88
.0177551207

0.92
.0186522793

0.96
.0201047459

1.
.0336670575

10000000.

0.04
.0003397118

0.08
.0011110725

0.12
.0018016232

0.16
.[illegible]24903148

0.2
.[illegible]551101

0.24
0.[illegible]348

0.28
.[illegible]

[illegible]

0.36
.0059039218

0.4
.0064477353

0.44
.0070719675

0.48
.0077049049

0.52
.0083939787

0.56
.009[illegible]834247

0.6
.00977[illegible]0436

0.64
0.010512957

0.68
.0112[illegible]

0.72
.0120999[illegible]8

0.76
.012[illegible]298657

0.8
.013[illegible]

0.84
.014[illegible]

0.88
.015[illegible]

0.92
.017[illegible]

0.96
.017[illegible]

1.
.[illegible]

August 20, 1980
BRP:pjk

30000000.

0.04
.0002938713

0.08
.0009611502

0.12
.0015585131

0.16
.0021547056

0.2
.0027302263

0.24
.0033075417

0.28
.0038914572

0.32
.0044569339

0.36
.0050202247

0.4
.0055776313

0.44
0.00611768

0.48
.0066652092

0.52
.0072612997

0.56
0.0078577[illegible]4

0.6
0.008472445

0.64
.[illegible]09199[illegible]6

0.68
[illegible]

0.72
.0104671632

0.76
.0111851329

0.8
.0119805112

0.84
.0126160882

0.88
.0130921054

0.92
.0137542697

0.96
.0148298736

1.
.0249206382

100000000.

0.04
.0002507068

0.08
.0008199744

0.12
.0013295954

0.16
.0018282179

0.2
.0023292149

0.24
0.0028217[illegible]3

0.28
.00331[illegible]718

0.32
.0038[illegible]52

0.36
.004[illegible]406

0.4
.0047584198

0.44
.0052191024

0.48
.0056862093

0.52
.0061947448

0.56
.00670355[illegible]4

0.6
.0072279945

0.64
.0077629758

0.68
.0083320659

0.72
.0089297243

0.76
0.009542298

0.8
.0102208738

0.84
.010753010[illegible]

0.88
.01116910[illegible]4

0.92
.01179401[illegible]

0.96
.0124[illegible]

1.
.0[illegible]26024[illegible]5

Lower Surface
$\alpha = 6.05^{\circ}$

300000.

0.04
.0014273985

0.08
.0019030852

0.12
.0022151029

0.16
.0026723149

0.2
.0029852357

0.24
.0032856392

0.28
.0035669673

0.32
.0038230785

0.36
0.004066352

0.4
.0042994953

0.44
.0045111377

0.48
.0047498256

0.52
.0049969619

0.56
.0052444108

0.6
.0055021377

0.64
0.0057774617
389.9904341
.0058914479

0.68
.0083474128

0.72
.0098021416

0.76
.0112887588

0.8
.0129327642

0.84
.0147683326

0.88
.0170621669

0.92
.0205865816

0.96
.0260615243

1000000.

0.04
.0007818183

0.08
.0010423627

0.12
.0012680341

0.16
.0014445[illegible]5

0.2
.0016349415

0.24
.0018012819

0.28
.0019527185
[illegible]
0.0021[illegible]

0.32
[illegible]

0.36
.0046423827

0.4
.0059601342

0.44
.0070141121

0.48
.0083771942

0.52
.0091406513

0.56
.0103014029

0.6
.0112813241

0.64
.0123[illegible]

0.68
.0135746298

0.72
.0148209049

0.76
.0161699429

0.8
.017844[illegible]

0.84
[illegible]

0.88
[illegible]

0.92
[illegible]

0.96
[illegible]

3000000.

0.04
0.000451383

0.08
.0006018084
303.5945488
.0007181982

0.12
.0018794981

0.16
0.002817789

0.2
.0036677105

0.24
.0045046659

0.28
.0053144198

0.32
.0061022753

0.36
.0068841857

0.4
.0076540251

0.44
0.008400148

0.48
0.009206618

0.52
.0100433745

0.56
.0108962655

0.6
.0117851952

0.64
0.01273319

0.68
.0137361872

0.72
.0148257537

0.76
.0160253321

0.8
.0175628167

0.84
.0198249493

0.88
.0226560227

0.92
.0268154896

0.96
.0345514573

10000000.

0.04
.0007563246

0.08
.0014063404

0.12
.0020720203

0.16
.0027359162

0.2
.0033614319

0.24
0.004047649

0.28
.0047071558

0.32
[illegible]

0.36
[illegible]

0.4
.0066457334

0.44
0.007263556

0.48
.0079492533

0.52
.0086574431

0.56
.0093807799

0.6
.0101365094

0.64
.0109342346

0.68
0.011800753

0.72
.0127327223

0.76
.0137604964

0.8
.0150620764

0.84
.0170330323

0.88
[illegible]

0.92
[illegible]

0.96
[illegible]

Upper Surface
$\alpha = -4.72^{\circ}$

30000000.

0.04
.0006542666

0.08
.0012165696

0.12
0.001792423

0.16
.0023567321

0.2
.0029260516

0.24
.0035031915

0.28
.0040724322

0.32
.0046326141

0.36
.0051936261

0.4
.0057489617

0.44
0.006299606

0.48
.0068766718

0.52
.0074892124

0.56
.0081149425

0.6
.0087686942

0.64
.0094592959

0.68
[illegible]

0.72
.0110145756

0.76
.0119036624

0.8
.0130469091

0.84
0.014734656

0.88
.0168492381

0.92
.0199407363

0.96
.0257563369

100000000.

0.04
.0005581665

0.08
.0010378773

0.12
.0015291481

0.16
.0020191024

0.2
0.002496267

0.24
.0029886554

0.28
.0034742557

0.32
0.003952156

0.36
.0044307754

0.4

0.44
.0053649234

0.48
.0058656118

0.52
.0063891612

0.56
.0069330027

0.6
0.00746073

0.64
0.008069893

0.68
.0087089395

0.72
.0093967316

0.76
.0101552275

0.8
.0111305517

0.84
.0125703968

0.88
.0143743166

0.92
.0170235615

0.96
.0219721921

Upper Surface
$\alpha = -4.72°$

B

0.72
.0340767333

0.76
.0365411337

0.8
.0393495444

0.84
.042031[illegible]

0.88
.0441123411

0.92
.0466144009

0.96
.0499865583

1000000.

0.04
.0002917821
119.3468967
.0003482138

0.08
.0026846164

0.12
0.004714378

0.16
.0065438078

0.2
0.008212802

0.24
.009[illegible]

0.28
.0113834564

0.32
.0128880596

0.36
.0140530431

300000.

A

0.04
.0005327188
65.36898749
.0006357467

0.08
.0033923696

0.12
0.005772202

0.16
.0079154906

0.2
0.009870125

0.24
.0117313082

0.28
.0135837779

0.32
.0153464849

0.36
.0170621393

0.4
0.018766551

0.44
.0204096396

0.48
.0[illegible]

0.52
.027[illegible]15083

0.56
.025[illegible]

[illegible]
.02[illegible]

0.6[illegible]
.029[illegible]

[illegible]

C

0.4
.0158073949

0.44
.0172106471

0.48
.0186013074

0.52
.0201195943

0.56
0.021708189

0.6
.0233391023

0.64
.0250526844

0.68
.0268796339

0.72
.0289705027

0.76
.0309721176

0.8
.0331668146

0.84
.0356537153

0.88
.0374227447

0.92
.0[illegible]

0.96
.04[illegible]

Lower Surface
$\alpha = -4.72^\circ$
(Read A→B→C)

3000000.

0.04
.0001684505
206.7148888
.0003010408

0.08
.0022201169

0.12
.0039761353

0.16
0.005559316

0.2
.0070038853

0.24
.0082793557

0.28
.0097475768

0.32
.0110503709

0.36
.0123179352

0.4
.0135770023

0.44
0.014790825

0.48
0.015994235

0.52
.0171251219

0.56
0.018682163

0.6
.0200930277

0.64
.0215753082

0.72
.0248774638

0.76
.0266951013

0.8
.0287560793

0.84
.0307439038

0.88
.0322792745

0.92
.0341248052

0.96
.03661132[illegible]7

10000000.

0.04
.0000922696
377.4060252
.0001101146

0.08
0.001834906

0.12
.0033332349

0.16
.0046842826

0.2
.0059171931

0.24
.0070910576

0.28
.0081567523

0.32
.0091703845

0.36
.0104320[illegible]6

0.4
.0115264672

0.44
.01256226[illegible]3

0.48
.0135891503

0.52
.0147246596

0.56
.0158824415

0.6
.0170860901

0.64
0.018350514

0.68
.0196996435

0.72
.0211674651

0.76
.0227179037

0.8
.0244842604

0.84
.0261714015

0.88
.0274811759

0.92
.0290554401

0.96
0.[illegible]

Lower Surface
$\alpha = -4.72°$

30000000.

0.04
.0000532719
653 6898749
.0000635747

0.08
.0015586066

0.12
.0028549159

0.16
.0040238703

0.2
.0050906206

0.24
.0061063472

0.28
.0071168042

0.32
.0080784205

0.36
.0090142965

0.4
.0099438414

0.44
0.010839396

0.48
0.011728431

0.52
0.012710767

0.56
.0137123576

0.6
.0147535842

0.64
.0158474914

0.68
.0170135461

0.72
.0182840755

0.76
.0196251926

0.8
.0211531246

0.84
0.022612384

0.88
.0237453808

0.92
.0251071091

0.96
.0269415679

100000000.

0.04
.0000291782
1190.468967
.0000348213

0.08
.0013131369

0.12
.0024191562

0.16
.0034165422

0.2
.0043257488

0.24
.0051934116

0.28
.0060555453

0.32
.0065720125

0.36
.0072745133

0.4
.0084675982

0.44
.0092021979

0.48
.0099902046

0.52
.0100282832

0.56
.0116527723

0.6
0.012571076

0.64
.0135043281

0.68
.0144990487

0.72
.0155829113

0.76
.0167269515

0.8
.0180303959

0.84
.0192752348

0.88
.0202417983

0.92
.0214034574

0.96
0.022369346

Lower Surface
$\alpha = -4.72^{\circ}$

```
057   16  A'
090   17  B'
119   12   B
136   18  C'
184   19  D'
253   10  E'
285   11   A
368   14   D


399.69

000   43  RCL
001   65   65
002   99  PRT
003   98  ADV
004   43  RCL
005   05   05
006   75   -
007   43  RCL
008   09   09
009   95   =
010   55   ÷
011   02   2
012   95   =
013   42  STO
014   01   01
015   42  STO
016   00   00
017   00   0
018   42  STO
019   04   04
020   42  STO
021   66   66
022   42  STO
023   02   02
024   43  RCL
025   09   09
026   85   +
027   09   9
028   95   =
029   32  X:T
030   01   1
031   00   0
032   67   EQ
033   14   D
034   01   1
035   75   -
036   43  RCL
037   10   10
038   95   =
039   45  Yˣ
040   02   2
041   93   .
042   05   5
043   95   =
044   42  STO
045   06   06
046   22  INV
047   86  STF
```

```
049   07   7
050   42  STO
051   64   64
052   01   1
053   01   1
054   42  STO
055   63   63
-------------
056   76  LBL
057   16  A'
058   01   1
059   75   -
060   73  RC*
061   63   63
062   95   =
063   42  STO
064   62   62
065   45  Yˣ
066   02   2
067   93   .
068   05   5
069   95   =
070   72  ST*
071   64   64
072   01   1
073   44  SUM
074   63   63
075   87  IFF
076   01   01
077   12   B
078   01   1
079   44  SUM
080   64   64
081   86  STF
082   01   01
083   43  RCL
084   63   63
085   42  STO
086   02   02
087   61  GTO
088   16  A'
-------------
089   76  LBL
090   17  B'
091   53   (
092   53   (
093   43  RCL
094   06   06
095   85   +
096   04   4
097   65   ×
098   43  RCL
099   07   07
100   85   +
101   43  RCL
102   08   08
103   54   )
104   65   ×
105   43  RCL
106   03   03
107   55   ÷
108   03   3
109   54   )
110   42  STO
```

```
111   04   04
112   44  SUM
113   66   66
114   07   7
115   42  STO
116   64   64
117   92  RTN
-------------
118   76  LBL
119   12   B
120   71  SBR
121   17  B'
122   22  INV
123   86  STF
124   01   01
125   43  RCL
126   02   02
127   22  INV
128   77   GE
129   18  C'
130   22  INV
131   67   EQ
132   18  C'
133   86  STF
134   02   02
-------------
135   76  LBL
136   18  C'
137   43  RCL
138   02   02
139   75   -
140   01   1
141   00   0
142   95   =
143   65   ×
144   43  RCL
145   03   03
146   95   =
147   99  PRT
148   53   (
149   43  RCL
150   66   66
151   55   ÷
152   53   (
153   43  RCL
154   62   62
155   45  Yˣ
156   03   3
157   54   )
158   54   )
159   34  √x
160   65   ×
161   05   5
162   95   =
163   55   ÷
164   53   (
165   43  RCL
166   65   65
167   34  √x
168   54   )
169   95   =
170   42  STO
171   67   67
172   99  PRT
```

```
173  87  IFF
174  02   02
175  19  D'
176  98  ADV
177  43  RCL
178  08   08
179  42  STD
180  06   06
181  61  GTD
182  16  A'
183  76  LBL
184  19  D'
185  43  RCL
186  62   62
187  34  ΓX
188  65   x
189  43  RCL
190  65   65
191  65   x
192  43  RCL
193  67   67
194  55   ÷
195  04   4
196  93   .
197  07   7
198  08   8
199  08   8
200  95   =
201  42  STD
202  02   02
203  99  PRT
204  65   x
205  05   5
206  93   .
207  07   7
208  01   1
209  04   4
210  55   ÷
211  53   (
212  43  RCL
213  65   65
214  65   x
215  43  RCL
216  62   62
217  34  ΓX
218  54   )
219  95   =
220  99  PRT
221  98  ADV
222  43  RCL
223  02   02
224  45  YX
225  01   1
226  93   .
227  01   1
228  05   5
229  02   2
230  95   =
231  42  STD
232  02   02
```

```
233  22  INV
234  86  STF
235  02   02
236  43  RCL
237  62   62
238  45  YX
239  01   1
240  93   .
241  06   6
242  05   5
243  95   =
244  42  STD
245  06   06
246  00   0
247  42  STD
248  66   66
249  07   7
250  42  STD
251  64   64
252  76  LBL
253  10  E'
254  01   1
255  75   -
256  73  RC*
257  63   63
258  95   =
259  42  STD
260  62   62
261  45  YX
262  01   1
263  93   .
264  06   6
265  05   5
266  95   =
267  42  STD
268  69   69
269  72  ST*
270  64   64
271  01   1
272  44  SUM
273  63   63
274  87  IFF
275  01   01
276  11   A
277  86  STF
278  01   01
279  01   1
280  44  SUM
281  64   64
282  61  GTD
283  10  E'
284  76  LBL
285  11   A
286  43  RCL
287  03   03
288  65   x
289  53   (
290  43  RCL
291  63   63
292  75   -
```

```
295  54   )
296  95   =
297  99  PRT
298  71  SBR
299  17  B'
300  43  RCL
301  66   66
302  65   x
303  43  RCL
304  65   65
305  55   ÷
306  08   8 .5
307  00   0 5
308  95   =
309  55   ÷
310  53   (
311  43  RCL
312  62   62
313  45  YX
314  01   1
315  93   .
316  01   1
317  05   5
318  54   )
319  95   =
320  85   +
321  43  RCL
322  02   02
323  95   =
324  22  INV
325  45  YX
326  01   1
327  93   .
328  01   1
329  05   5
330  02   2
331  95   =
332  65   x
333  05   5
334  93   .
335  07   7
336  01   1
337  04   4
338  55   ÷
339  53   (
340  43  RCL
341  65   65
342  65   x
343  53   (
344  43  RCL
345  62   62
346  34  ΓX
347  54   )
348  54   )
349  95   =
350  99  PRT
351  98  ADV
352  22  INV
353  86  STF
```

355	43	RCL
356	69	69
357	42	STD
358	06	06
359	97	DSZ
360	00	00
361	10	E'
362	43	RCL
363	01	01
364	42	STD
365	00	00
366	91	R/S
367	76	LBL
368	14	D
369	00	0
370	42	STD
371	06	06
372	07	7
373	42	STD
374	64	64
375	01	1
376	01	1
377	42	STD
378	63	63
379	61	GTD
380	10	E'
381	00	0
382	00	0
383	00	0
384	00	0
385	00	0

12.	00
12.	01
977.3428482	02
0.02	03
.0335920914	04
49.	05
.7456336761	06
.8404774784	07
.7456336761	08
25.	09
1.	10
0.4290993	11
-0.1488981	12
-0.1738043	13
-0.2042837	14
-0.2297392	15
-0.2494459	16
-0.2646265	17
-0.277503	18
-0.2902193	19
-0.3021669	20
-0.3112488	21
-0.3171539	22
-0.3218098	23
-0.3263865	24
-0.3312559	25
-0.3353662	26
-0.3389635	27
-0.3420992	28
-0.344265	29
-0.3462343	30
-0.3490858	31
-0.351841	32
-0.3531389	33
-0.352087	34
-0.3489647	35
-0.3453159	36
-0.3415661	37
-0.3369379	38
-0.331439	39
-0.3251038	40
-0.3177958	41
-0.309659	42
-0.3003378	43
-0.289526	44
-0.2774239	45
-0.2647698	46
-0.2506704	47
-0.235302	48
-0.2181101	49
-0.1971979	50
-0.1685343	51
-0.1328144	52
-0.09420395	53
-0.00978433	54
-0.9378433	55
0.04223716	56
0.09996742	57

0.	61
0.8370033	62
59.	63
7.	64
300000.	65
.7160031549	66
.0054082583	67
977.3428482	68
.7456336761	69

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B: Perturbed Pressure Distributions Caused by Surface Irregularities

In this discussion we include two bumps because they lead to the simplest computations. The bumps presented are:

1st case: $\eta(x) = \varepsilon(1 - 4\,x^2) \quad , \quad |x| \le 1/2 \quad ; \quad = 0 \quad , \quad |x| > 1/2$.

2rd case: $\eta(x) = \varepsilon(1 - 4\,x^2)^2 \quad , \quad |x| \le 1/2 \quad ; \quad = 0 \quad , \quad |x| > 1/2$.

For Case 1: we have

$$\eta'(x) = -8\varepsilon x \quad , \quad |x| \le 1/2 \quad ; \quad \eta'(x) = 0 \quad \text{elsewhere} \quad .$$

$$\eta''(x) = -8\varepsilon \quad , \quad |x| \le 1/2 \quad ; \quad \eta''(x) = 0 \quad \text{elsewhere} \quad .$$

For Case 2: we have

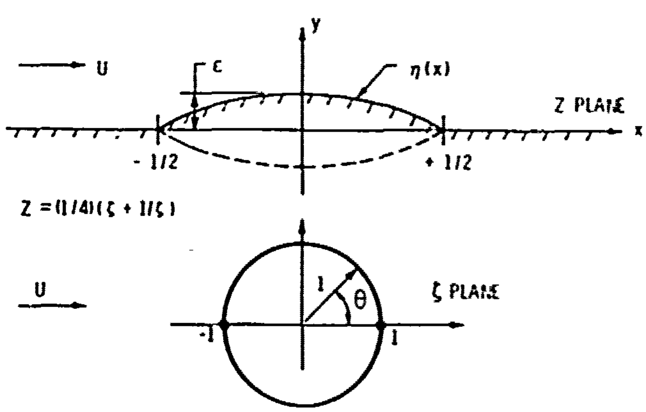
$$\eta'(x) = -16\varepsilon x(1 - 4\,x^2) \quad , \quad |x| \le 1/2 \quad ; \quad \eta'(x) = 0 \quad \text{elsewhere}$$

$$\eta''(x) = -16\varepsilon\,(1 - 12\,x^2) \quad , \quad |x| \le 1/2 \quad ; \quad \eta''(x) = 0 \quad \text{elsewhere}$$

Note that both bumps have discontinuous curvature at $|x| = 1/2$ but that in Case 2 the slope is continuous. We have considered bumps having a continuous slope and curvature at $|x| = 1/2$ but the series convergence is not too good.

Case 1

Consider the bump for which $\eta'(x) = -8\varepsilon x$ as noted above.



In accordance with the ideas of thin airfoil theory we consider the <u>complex</u> velocity, $W(z) = U(1 + u - iv)$, to be invariant at corresponding points in the z and ζ planes. Next we consider the mapping,

$$z = (x + iy) = \frac{1}{4}\left(\zeta + \frac{1}{\zeta}\right) \quad .$$

on the unit circle, $\zeta = e^{i\theta}$, and $2x = \cos\theta$. Accordingly, $\eta'(x) = -4\varepsilon\cos\hat{\theta}$. But in accordance with the scheme of linearization the boundary condition, $v(x)/U = \eta'(x)$, is to be evaluated at $y = 0$ on the real axis. Thus we have

$$\operatorname{Im}\left[\frac{W(x)}{U}\right] = \begin{cases} \eta'(x) \quad , & |x| \leq \frac{1}{2} \quad \text{on the bump} \quad , \\ \\ 0 \quad , & |x| \leq \frac{1}{2} \quad \text{off the bump} \quad . \end{cases}$$

But we know that for flows without circulation

$W(z) = U[1 + w(z)]$ can be written with

$$w(z) = \sum_{i=1}^{\infty} \frac{a_i + ib_i}{\zeta^i} \qquad \text{(where } w = u - iv \text{ is the perturbation velocity)}$$

in order that $w(z) \to 0$ as $\zeta \to \infty$ and that the flow direction at ∞ will be invariant under the mapping. This latter aspect is accounted for in the Joukowski transformation.

In general terms, $\zeta = re^{i\theta}$. But on the unit circle

$$\zeta = e^{i\theta} \quad .$$

Therefore, the real axis in the interval, ($y = 0$, $|x| \leq 1/2$) is on the unit circle with $0 \leq \theta \leq \pi$. The point ($y = 0$, $x = 0$) and at the center of the bump, where $\eta'(0) = 0$ and $\eta(0) = \varepsilon$, maps into the point $\theta = \pi/2$, $r = 1$. Accordingly, we have

$$w = u - iv = \sum_{i=1}^{\infty} (a_i + ib_i)e^{-i\theta} = \sum_{i=1}^{\infty} (a_i \cos i\theta + b_i \sin i\theta)$$

$$- i \sum_{i=1}^{\infty} (a_i \sin i\theta - b_i \cos i\theta) \quad .$$

Applying the boundary conditions on the bump we have

$$-iv = -i \sum_{i=1}^{\infty} (a_i \sin i\theta - b_i \cos i\theta) = +i8\varepsilon x, \ |x| \leq \frac{1}{2} = 0, \ |x| > \frac{1}{2}$$

Therefore in the ζ plane we have

$$v = \sum_{i=1}^{\infty} (a_i \sin i\theta - b_i \cos i\theta) = -4\varepsilon \cos\theta \text{ on } \zeta = e^{i\theta} \quad .$$

But if $v = 0$ at points on the real axis outside the unit circle and on $|\zeta| = 1$ if $v(-\theta) = -v(\theta)$, in order that $v(\theta)$ is odd with respect to θ so that it will represent a thickness distribution and <u>not</u> a camber function, we must consider the expansion of $\cos\theta$ as a sine series. In this case $b_i = 0$ and

$$a_i = \frac{-8\varepsilon}{\pi} \int_0^{\pi} (\cos\theta) \sin i\theta \, d\theta = \begin{cases} \dfrac{-16\varepsilon i}{\pi(i^2-1)} & , \quad i = 2, 4, 6, \ldots, \\ 0 & , \quad \text{otherwise} \end{cases} \quad .$$

As a result of this finding it turns out that the u component is given by

$$u = -\frac{16\varepsilon}{\pi}\sum_{i=2}^{\infty}\frac{i\cos i\theta}{(i^2-1)} \quad , \quad \text{(i is even only)}$$

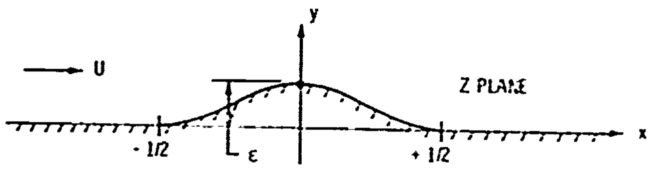
and since the linearized bernoulli equation is $C_p = -2u$, we have

$$C_p = \frac{32\varepsilon}{\pi}\sum_{i=2\ldots}^{\infty}\frac{i\cos i\theta}{i^2-1} \quad ; \quad \text{on} \quad \zeta = e^{i\theta} \quad , \text{ and } i \text{ even} \quad .$$

At this point we need to pause and consider the convergence of $C_p(\theta)$. In accordance with the Laurent expansion for w given on page 59 it is clear that for points outside and on the circle we must have $\zeta = re^{i\theta}$ and

$$C_p(\zeta) = \frac{32\varepsilon}{\pi}\sum_{i=2}^{\infty}\frac{i\cos i\theta}{r^i(i^2-1)} \quad , \quad \text{with } i \text{ being even} \quad .$$

But, on the circle $r = 1$ and on the real axis $r \geq 1$ and $\theta = 0, \pi$. In particular, if $r > 1$ the series converges for all θ. When $r = 1$ the series diverges at $\theta = 0, \pi$. This can be verified by employing the usual tests for convergence of power series. It can also be seen for sufficiently large values of i when $\theta = 0$ (or π) that the series becomes equivalent to the <u>harmonic series</u> which is known to be divergent. Therefore we see that in terms of thin airfoil theory we can consider the corner between the wall and the bump as a linearized <u>stagnation point</u>. Since the series at other points is convergent, we can compute values of C_p with it, but we should not expect rapid convergence. Indeed, as we shall see below even though we take many terms (as high as 40) the accuracy is not great.



Case 2: $\eta'(x) = -16\ \varepsilon x\ (1 - 4x^2) = -8\ \varepsilon \cos\theta \sin^2\theta$

$$= -4\ \varepsilon \cos\theta \sin^2\theta = -2\ \varepsilon\ (\cos\theta - \cos 3\theta)$$

Note that this case contains the first case as part of the solution. Thus we see that the term $-2\varepsilon \cos\theta$ will lead to a contribution for $u(\theta)$ which is

$$-\frac{8\varepsilon}{\pi} \sum_{i=2}^{\infty} \frac{i \cos i\theta}{(i^2-1)} \quad , \quad i = 2, 4, \ldots \quad .$$

It remains for us to evaluate the term $2\varepsilon \cos 3\theta$. In this instance we have to consider

$$+\frac{4\varepsilon}{\pi} \int_0^{\pi} \cos 3\theta \sin i\theta \, d\theta = \begin{cases} \dfrac{8\varepsilon i}{\pi(i^2-9)} & , \quad n = 2, 4, 6, \ldots \ , \\ 0 & , \quad \text{otherwise} \end{cases} \quad .$$

If we add these results, term by term, we obtain

$$\frac{8\varepsilon}{\pi}\left[\frac{i}{i^2-9}-\frac{i}{i^2-1}\right]=\frac{8\varepsilon}{\pi}\left[\frac{i^3-i-i^3+9i}{(i^2-1)(i^2-9)}\right]=\frac{64\varepsilon}{\pi}\,\frac{i}{(i^2-1)(i^2-9)}$$

Therefore we have

$$C_p(x)=-\frac{128\varepsilon}{\pi}\sum_{i=2}^{\infty}\frac{i\cos i\theta}{r^i(i^2-1)(i^2-9)}=\frac{32\varepsilon}{\pi}\sum_{i=2}^{\infty}\left\{\frac{i\cos i\theta}{r^i(i^2-1)}\left(\frac{4}{9-i^2}\right)\right\}\quad,$$

where $i = 2, 4, 6 \ldots$, as before.

Note that we have written this expression so that each term of the sum for C_p can be obtained from the corresponding term of the first case by multiplying it by the factor $4/(9-i^2)$.

The convergence of this series is no problem. As $i \to \infty$ each term approaches $1/i^2$, and it is known that this will produce a uniformly and absolutely convergent series both on and off the circle $r = 1$.

Similarly, it is worth noting that the formulae for $\eta(x)$ in the two cases are very simply related. In fact, if we want to get case 2 from case 1 we need only take $\eta(x)/\varepsilon$ for case 1 and square it. As a result it is fairly easy to program a routine which will do both cases in sequence and this has been done below.

It should be noted that the 2nd case was computed on the basis that the multiplicative factor $4/(9-i^2)$ was $2/(9-i^2)$, and this is incorrect. Therefore we should multiply all results for $C_p(x)/\varepsilon$ which are tabulated below by a factor of 2. This error was corrected in the case $N = 40$.

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Notes on T.I. - 59 program for finding $C_p(x)/\varepsilon$ for the two bumps:

Case 1: $\eta(x) = \varepsilon(1 - 4x^2)$,

Case 2: $\eta(x) = \varepsilon(1 - 4x^2)^2$.

Procedure: Enter program from keyboard. See below.

Enter Input Data:	Data Register Address
Total number of terms in sum = N	R_{01} → |STØ| 01
Increment between successive x = Δx	R_{07} → |STØ| 07
Maximum value of x = x_{max}	R_{08} → |STØ| 08

In order to execute program after input data are entered:

Press: |CLR|

|RST|

|R/S|

If it is desired to run Case 2 only, enter input data as before. Then

Press: |CLR|

|2nd| |St flg|

|GTO| |03|

|R/S|

Storage Map:

R_{00} for DSZ(n)	R_{04} for r(x)	*R_{08} for $x\|_{max}$
*R_{01} for N	R_{05} for i	R_{09} for $C_p(x)\varepsilon$
R_{02} for x	R_{06} for $\eta(x)/\varepsilon$	R_{10} for (x-1/2)
R_{03} for θ	*R_{07} for Δx	R_{11} used in summation of $C_p(x)$.

*Input quantities.

—//—

Data calculated for two cases, N = 20 and N = 40 are given below. These are followed by the program listing and other information. A plot of Case 1 and Case 2 is given in Figure 3, page 26 above.

$N = 40$
Case 1.
$\eta = \epsilon(1-4x^2)$

0.
1.
-5.030082152

0.05
0.99
-5.058153468

0.1 x
0.96 η/ϵ
-4.939254581 C_p/ϵ

0.15
0.91
-4.56[illegible]662501

0.2
0.84
-4.253528991

0.25
0.75
-3.693644285

0.3
0.64
-2.997821768

0.35
0.51
-1.912952636

0.4
0.36
-.5158775378

0.45
0.19
1.521340967

0.5
0.
23.7574337

0.55
0.
3.435135819

0.6
0.
2.234470026

0.65
0.
1.649982227

1.4
0.
0.2347862

1.45
0.
.2176112402

1.5
0.
.2022962247

1.55
0.
.1885765801

1.6
0.
.1762339944

1.65
0.
.1650868878

1.7
0.
.1549831282

1.75
0.
.1457944004

1.8
0.
.1374118072

1.85
0.
0.129742402

1.9
0.
0.122706431

1.95
0.
0.116235122

2.
0.
.110268[illegible]982

0.7
0.
1.294791052

0.75
0.
1.054641806

0.8
0.
.8814365151

0.85
0.
0.750894088

0.9
0.
.6492842871

0.95
0.
.5682095794

1.
0.
.5022282621

1.05
0.
.4476533426

1.1
0.
0.401896424

1.15
0.
.36308634[illegible]4

1.2
0.
.3292394981

1.25
0.
.3011075[illegible]1

1.3
0.
.2[illegible]608560[illegible]5

1.35
0.
[illegible]

$N = 40$ Case 2 $\eta = \epsilon(1-4x^2)^2$

0.
1.
-6.790649262

0.05 x
0.9801 η/ϵ
-6.58756297 C_p/ϵ

0.1
0.9216
-5.986658923

0.15
0.8281
-5.013216552

0.2
0.7056
-3.710924596

0.25
0.5625
-2.145937846

0.3
0.4096
-.4124959147

0.35
0.2601
1.354261938

0.4
0.1296
2.951140955

0.45
0.0361
4.024363908

0.5
0.
3.396858894

0.55
0.
1.952548409

0.6
0.
1.42897183

0.65

0.7
0.
.9093066337

0.75
0.
.7587009382

0.8
0.
.6452235254

0.85
0.
0.5569258

0.9
0.
.4865118466

0.95
0.
.4292514483

1.
0.
.3819358799

1.05
0.
.3423096564

1.1
0.
.3087409166

1.15
0.
.2800203531

1.2
0.
0.255233431

1.25
0.
.2336763757

1.3
0.
.2147993228

1.35
0.

1.4
0.
0.183430237

1.45
0.
.1703068729

1.5
0.
.1585658571

1.55
0.
.1480167431

1.6
0.
.1385012363

1.65
0.
.1298868128

1.7
0.
.1220617842

1.75
0.
.1149314434

1.8
0.
.1084150255

1.85
0.
.1024432906

1.9
0.
.0969565865

1.95
0.
.0919032844

2.
0.
.0872385066

No. of terms in sum = N = 20

0. x
1. η/ϵ
-4.968739687 C_p/ϵ

Case 1

$\eta = \epsilon(1-4x^2)$

0.05
0.99
-5.112666205

0.1
0.96
-4.936602309

0.15
0.91
-4.490128291

0.2
0.84
-4.28217104

0.25
0.75
-3.819338671

0.3
0.64
-2.92673393

0.35
0.51
-1.840841503

0.4
0.36
-.4227985398

0.45
0.19
1.542741737

0.5
0.
20.28900423

0.55
0.
3.435135815

0.6
0.
2.234470026

0.65
0.
1.649982227

0.7
0.
1.294791052

0.75
0.
1.054641806

0.8
0.
.8814365151

0.85
0.
0.750894088

0.9
0.
.6492842871

0.95
0.
.5682095794

1.
0.
.5022282621

1.05
0.
.4476533426

1.1
0.
0.401896424

1.15
0.
.3630868414

1.2
0.
.3298394981

1.25
0.
.3011075311

1.3
0.
[illegible]

1.35
0.
.2541445596

1.4
0.
0.2347862

1.45
0.
.2176112402

1.5
0.
.2022962247

1.55
0.
.1865765801

1.6
0.
.1762339944

1.65
0.
.1650868878

1.7
0.
.1549831282

1.75
0.
.1457944004

1.8
0.
.1374118072

1.85
0.
0.129742402

1.9
0.
0.122706431

1.95
0.
0.116235122

2.
0.
[illegible]

No of terms in sum $N=20$

Case 2

$\eta = \epsilon(1-4x^2)^2$

x	η/ϵ	C_p/ϵ
0.	1.	-3.395453596
0.05	0.9801	-3.293702658
0.1	0.9216	-2.993284339
0.15	0.8281	-2.506745124
0.2	0.7056	-1.855410596
0.25	0.5625	-1.072817063
0.3	0.4096	-.2063038399
0.35	0.2601	.6769714676
0.4	0.1296	1.475377376
0.45	0.0361	2.012301881
0.5	0.	1.700689672
0.55	0.	.9762742044
0.6	0.	.7144859148
0.65		
0.7	0.	.4546533169
0.75	0.	.3793504691
0.8	0.	.3226117627
0.85	0.	0.2784629
0.9	0.	.2432559233
0.95	0.	.2146257241
1.	0.	0.19096794
1.05	0.	.1711548282
1.1	0.	.1543704583
1.15	0.	.1400101765
1.2	0.	.1276167155
1.25	0.	.1168381879
1.3	0.	.1073996619
1.35	0.	.0990834462
1.4	0.	.0917151185
1.45	0.	.0851534364
1.5	0.	.0792829285
1.55	0.	.0740083715
1.6	0.	.0692506182
1.65	0.	.0649434064
1.7	0.	.0610308921
1.75	0.	.0574657217
1.8	0.	.0542075128
1.85	0.	.0512216453
1.9	0.	.0484782933
1.95	0.	.0459516422
2.	0.	

All values of $C_p(x)/\epsilon$ on this page should be multiplied by 2 in order to correct an error in the program.

BRP:pjk
August 20, 1980

DATA REGISTERS

Contents	Variable name	Register
17.	for DSZ	00
40.	N	01
0.35	x	02
.7953988302	θ	03
1.	r	04
2.	i	05
0.2601	$\eta(x)/\epsilon$	06
0.05	Δx	07
2.	x_{max}	08
.0664847757	$C_p(x)/\epsilon$	09
-0.15	$(x-1/2)$	10
-.0133333333	USED	11
0.		12
0.		13
0.		14
0.		15
0.		16
0.		17
0.		18
0.		19
0.		20
0.		21
0.		22
0.		23

contents ↑ variable name ↑ address ↑ — not used (registers 12–23)

Labels used
OP 08

041	13	C
068	15	E
071	16	A'
101	48	EXC
107	17	B'
129	18	C'
148	19	D'
154	14	D
195	11	A
221	12	B
242	10	E'
265	66	PAU

Program Listing

Loc	Code	Key	Comment
000	22	INV	Initialize main variables. Flag =1
001	86	STF	
002	01	01	
003	00	0	First x
004	42	STO	
005	02	02	
006	43	RCL	for DSZ
007	01	01	
008	42	STO	
009	00	00	
010	00	0	for C_p sum
011	42	STO	
012	09	09	
013	02	2	first i in sum
014	42	STO	
015	05	05	
016	43	RCL	Tests for Program Direction
017	02	02	
018	75	-	$(x-\frac{1}{2})$ for test.
019	93	.	
020	05	5	
021	95	=	
022	42	STO	
023	10	10	
024	43	RCL	Test on
025	08	08	
026	75	-	
027	43	RCL	$(x_{max}-x)$
028	02	02	
029	95	=	
030	77	GE	$x_{max} \geq x$?
031	13	C	yes
032	87	IFF	no — is flag 1 set?
033	01	01	
034	15	E	yes — exit.
035	86	STF	no — set Flg 1
036	01	01	
037	61	GTO	Start next case
038	00	00	
039	03	03	
040	76	LBL	Main Program
041	13	C	
042	43	RCL	Print x
043	02	02	
044	99	PRT	
045	71	SBR	calculate η/ϵ & $r(x)$
046	16	A'	
047	43	RCL	Print $\eta(x)/\epsilon$
048	06	06	
049	99	PRT	
050	71	SBR	calculate C_p/ϵ
051	18	C'	
052	43	RCL	Print $\left(\frac{C_p(x)}{\epsilon}\right)$
053	09	09	
054	99	PRT	
055	98	ADV	
056	43	RCL	
057	02	02	
058	85	+	
059	43	RCL	Δx
060	07	07	
061	95	=	$x+\Delta x$
062	42	STO	new x
063	02	02	
064	61	GTO	Start on next x calc.
065	00	00	
066	06	06	
067	76	LBL	end calculation
068	15	E	exit
070	76	LBL	Subroutine A'
071	16	A'	
072	70	RAD	
073	01	1	Set $r=1$
074	42	STO	
075	04	04	
076	00	0	Set $\theta=0$
077	42	STO	
078	03	03	
079	43	RCL	
080	10	10	is
081	77	GE	$x \geq \frac{1}{2}$?
082	17	B'	yes
083	53	(	no — when we calculate $\eta/\epsilon = 1-4x^2$ first & $(1-4x^2)^2$ 2nd, we must have $r=1$, θ is calculated below in Subroutine C'.
084	01	1	
085	75	-	
086	04	4	
087	65	×	
088	53	(	
089	43	RCL	
090	02	02	
091	33	X^2	$(1-4x^2)$
092	54	)	
093	54	)	
094	42	STO	First $\eta(x)/\epsilon$
095	06	06	
096	87	IFF	is flg 1 set?
097	01	01	
098	48	EXC	yes
099	92	RTN	no
100	76	LBL	flg 1 is set
101	48	EXC	
102	33	X^2	2nd
103	42	STO	$\eta(x)/\epsilon = (1-4x^2)^2$
104	06	06	
105	92	RTN	
106	76	LBL	$x \geq \frac{1}{2}$ — when we must calculate $r(x)$, $x \geq \frac{1}{2}$, we must have $\eta(x)=0$, $\theta=0$.
107	17	B'	
108	53	(	
109	43	RCL	
110	02	02	x
111	65	×	
112	02	2	2x
113	85	+	
114	53	(	
115	24	CE	
116	33	X^2	$(2x)^2$
117	75	-	
118	01	1	
119	54	)	
120	34	$\sqrt{x}$	$\sqrt{(2x)^2-1}$
121	54	)	
122	42	STO	$r=2x+\sqrt{\ }$
123	04	04	
124	00	0	
125	42	STO	$\eta(x)/\epsilon = 0.$
126	06	06	
127	92	RTN	

Subroutine C'

calculate θ if $x<\frac{1}{2}$

calculate terms in Sum for Cp(x).

... for first η(x).

128	76	LBL	
129	18	C'	
130	70	RAD	
131	43	RCL	
132	10	10	is $x\geq\frac{1}{2}$
133	77	GE	?
134	19	D'	yes
135	53	(	no
136	53	(	
137	43	RCL	
138	02	02	x
139	65	×	
140	02	2	
141	54	)	2x
142	22	INV	
143	39	COS	
144	54	)	
145	42	STO	θ
146	03	03	
147	76	LBL	$x\geq\frac{1}{2}$
148	19	D'	
149	43	RCL	is
150	10	10	$x=\frac{1}{2}$
151	67	EQ	?
152	66	PAU	yes
153	76	LBL	no
154	14	D	
155	53	(	
156	53	(	
157	43	RCL	i
158	05	05	
159	65	×	
160	53	(	
161	53	(	
162	24	CE	ε
163	65	×	
164	43	RCL	i
165	03	03	
166	54	)	
167	39	COS	
168	54	)	cos iθ
169	54	)	i cos iθ
170	55	÷	
171	53	(	
172	53	(	
173	43	RCL	r
174	04	04	
175	45	Y^X	
176	43	RCL	i
177	05	05	
178	54	)	r^i
179	65	×	
180	53	(	
181	43	RCL	
182	05	05	i^2
183	33	X^2	
184	75	-	
185	01	1	
186	54	)	i^2-1
187	54	)	$r^i(i^2-1)$

For case 1 or case 2 accumulate sum of terms

Test

sum is complete. calculate Cp(x)/ε.

Sum is not complete calculate next value of i for calculation of next term in Cp(x)/ε.

191	87	IFF	flg 1
192	01	01	set ?
193	10	E'	yes
194	76	LBL	no
195	11	A	
196	53	(	
197	43	RCL	term from first case
198	11	11	or 2nd case
199	85	+	
200	43	RCL	
201	09	09	Previous sum
202	54	)	
203	42	STO	new sum
204	09	09	is sum
205	97	DSZ	complete
206	00	00	?
207	12	B	no
208	53	(	yes
209	43	RCL	
210	09	09	
211	65	×	
212	03	3	
213	02	2	
214	55	÷	
215	89	π	
216	54	)	
217	42	STO	$\frac{Cp(x)}{\epsilon}$
218	09	09	
219	92	RTN	
220	76	LBL	
221	12	B	
222	53	(	
223	53	(	
224	43	RCL	N
225	01	01	
226	75	-	
227	53	(	
228	43	RCL	n from
229	00	00	DSZ
230	75	-	
231	01	1	
232	54	)	
233	54	)	
234	65	×	
235	02	2	
236	54	)	new value
237	42	STO	of i
238	05	05	
239	61	GTO	
240	18	C'	

calculate term in Sum for 2nd η(x)

if $x=\frac{1}{2}$ make $\theta=0$.

241	76	LBL	
242	10	E'	
243	53	(	
244	43	RCL	
245	11	11	
246	65	×	
247	53	(	
248	02	~~2~~ 4	
249	94	+/-	
250	55	÷	
251	53	(	
252	43	RCL	
253	05	05	
254	33	X^2	
255	75	-	
256	09	9	
257	54	)	
258	54	)	
259	54	)	
260	42	STO	
261	11	11	
262	61	GTO	
263	11	A	
264	76	LBL	
265	66	PAU	
266	00	0	Set
267	42	STO	θ=
268	03	03	0.
269	61	GTO	
270	14	D	
271	00	0	
272	00	0	
273	00	0	
274	00	0	

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